Explorations in Modulation Formats for Nonlinear DWDM Fiber Optic Communications

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Chapter 1

Introduction

1.1 Background & Motivation

The importance of fiber optics in contemporary high speed digital communications cannot be overestimated. With the increasing worldwide popularity of the Internet and numerous voice and video communication systems the demand for faster data rates will likely accelerate. Total Internet traffic in the United States alone is expected to reach 35 terabits per second by the end of the year [1]. Throughout the latter quarter of the 20th century fiber optics has been indispensable in facilitating this extraordinary load.

The theoretical potential of fiber optic communications has been known for several decades. It was not until the mid-1970’s however, with the advent of low-loss fibers and single-frequency light sources, that fiber optic communications became a practical solution [2, 3]. Subsequently there has been an explosion of interest in fiber optics. Contemporary optical fibers have a bandwidth in excess of 30 terahertz [4]. Modern commercial fiber optic systems are capable of transmitting hundreds of gigabits-per-second, with experimental systems demonstrating terabit capability [3]. Predictions are already being made that tens-of-terabit systems are on the not too distant horizon [4].

While the fiber channel may be capable of transmitting terabit-per-second (Tb/s) data rates, no current single electrical communication system can make complete use of this speed. The electrical transmitters and receivers on either end of a fiber channel are sub-
ject to computational constraints. Modern commercial systems are capable of operating at 10 gigabits-per-second (Gb/s) with experimental systems clocking 40 Gb/s performance [1].

Wavelength-division multiplexing (WDM) has proven to be a powerful solution to this disparity [3, 4, 5]. In WDM systems the available bandwidth is divided into separate channels with each channel carrying one signal. The data rate of each channel can be limited, frequently to 10 Gb/s, but with many channels the total data rate is high.

WDM has not always been a popular choice. The invention of erbium-doped fiber amplifiers (EDFA) is largely responsible for enabling this technique [3]. In order for WDM to be useful not only must the fiber provide a large bandwidth, but so must the optical amplifiers. The bandwidth limitations of previous amplifiers had limited WDM’s potential.

Other techniques have been explored for fully exploiting the broad bandwidth of fiber. These include time-division multiplexing (TDM) and, in multi-user situations, optical code-division multiple access (OCDMA). While these techniques are effective in many applications, they do not alleviate the problem outlined above. At this time WDM is the technique of choice for many fiber optic communication systems [1, 3, 5, 6, 7].

WDM has found convenient application in three areas, high-capacity point-to-point links, broadcast networks and multiple-access networks [3]. A typical example of a high-capacity point-to-point link would be a trans-oceanic fiber link. That is, a system with a single user transmitting a large data rate over a long distance. In contrast, a broadcast network is typically shorter distance with a single transmitter and numerous scattered receivers. An example of this would be a cable TV distribution network. A multiple-access network consists of numerous users all transmitting and receiving data, often asynchronously as with the Internet. Of particular interest for this thesis is the high-capacity point-to-point scenario, although the multiple-access network will be considered briefly.

Two of the primary limiting factors in long-haul WDM systems are dispersion and
fiber nonlinearities. Prior to the 1990’s dispersion was typically viewed as the larger obstacle, and nonlinear effects were frequently neglected. However, with improvements in dispersion shifted fibers, dispersion compensation filters and other dispersion management techniques, the problem of fiber dispersion has been somewhat mitigated. Simultaneously, as data rates have continued to increase, fiber nonlinearities have recently appeared as the most important limiting factor [8, 9, 10, 11]. Of the numerous nonlinearities that fiber optic systems are subject to, four-wave mixing (FWM) and cross-phase modulation (XPM) have been found to be the most destructive. FWM is frequently the more problematic of the two for dense WDM (DWDM) systems [9, 12, 13, 14, 15].

The intention of this thesis is to explore several modulation techniques with the hope of alleviating degradation due to four-wave mixing and cross-phase modulation in high-capacity long-haul DWDM systems.

The long-haul scenario is unique in that it simultaneously requires high data rates, high power levels and long distances. This is precisely the condition under which nonlinear effects are the limiting factor. In spite of the interest that fiber nonlinearities have garnered throughout the last two decades, there is still a great deal about these effects that is unknown. The philosophy behind this thesis is to apply several classical communication theory techniques to the relatively unexplored domain of nonlinear fiber optics, and see which techniques yield promising results.

The next section presents the fundamental equation that governs wave propagation in single mode fibers. Section 1.3 provides a brief description of the most important dispersive effects in fiber optic communications. A more thorough description of fiber nonlinearities is given in Chapter 2. Chapter 3 describes the simulation model, including an outline of the split-step Fourier method. Chapters 4, 5 and 6 each describe the theory and simulation results of one of the modulation techniques explored. Chapter 7 presents conclusions and recommendations for future work.
1.2 The Propagation Equation

The behavior of a pulse propagating in an optical fiber has been studied extensively [2, 3, 8] and distilled into the following variant of the nonlinear Schrödinger equation (NLS).

Consider the complex envelope \( A = A(t, z) \) of the normalized electric field \( E = E(t, z) \), where \( t \) is time and \( z \) is distance. \( A(t, z) \) is defined as

\[
E(t, z) = \frac{1}{2} \left( A(t, z)e^{i\omega_c t} + A^*(t, z)e^{-i\omega_c t} \right).
\]  

(1.1)

where \( A^* \) is the complex conjugate of \( A \) and \( \omega_c \) is the carrier frequency. A well known approximation for field propagation in single-mode fibers is [8]

\[
\frac{\partial A}{\partial z} + \frac{i\alpha}{2} A + i\beta_2 \frac{\partial^2 A}{\partial T^2} - \beta_3 \frac{\partial^3 A}{\partial T^3} = i\gamma \left( |A|^2 A + \frac{i}{\omega_c} \frac{\partial}{\partial T} (|A|^2 A) - T_R A \frac{\partial |A|^2}{\partial T} \right)
\]  

(1.2)

where \( T \) is the so-called retarded frame of reference, \( T = t - \frac{z}{v_g} \), \( v_g \) is the group velocity, \( \alpha \) is the attenuation coefficient and \( T_R \) represents the contribution due to Raman scattering. The dispersion terms, \( \beta_2 \) and \( \beta_3 \), will be described in Section 1.3. The nonlinear coefficient, \( \gamma \), is given by [8]

\[
\gamma = \frac{\bar{n}_2 \omega_c}{c A_{\text{eff}}} = \frac{2\pi \bar{n}_2}{\lambda A_{\text{eff}}}
\]  

(1.3)

where \( \bar{n}_2 \) is the nonlinear index coefficient of the cladding material, \( c \) is the speed of light, \( \lambda \) is wavelength and \( A_{\text{eff}} \) is the effective cross-sectional area of the fiber.

As mentioned in Section 1.1, modern WDM systems operate with individual channel data rates on the order of 10 Gb/s. This corresponds to pulse widths on the order of 100 picoseconds (ps). For pulse widths greater than 5 picoseconds the \( \frac{1}{\omega_c} \), \( T_R \) and \( \beta_3 \) terms are very small and hence can be ignored. This leaves [8]

\[
i \frac{\partial A}{\partial z} + i\frac{\alpha}{2} A - \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} + \gamma |A|^2 A = 0
\]  

(1.4)

(1.4) is often referred to as the nonlinear Schrödinger equation, and (1.2) is referred to as the generalized NLS equation. A direct solution for (1.4) has not yet been found, although an approximate closed-form solution using the Volterra series transfer function was
found in [16]. Nonetheless, insight into the NLS equation can be had by considering the dispersion and nonlinear terms separately.

1.2.1 The Dispersion Term

Ignoring the nonlinear term, (1.4) becomes [8, 17]

\[
\frac{\partial A}{\partial z} = -\frac{\alpha}{2} A - \frac{i \beta_2}{2} \frac{\partial^2 A}{\partial T^2}.
\] (1.5)

This is a differential equation in two domains, time and space. Taking the Fourier transform over time of (1.5) removes the derivative in the time domain. Let \( \tilde{A} = \tilde{A}(\omega, z) \) be the Fourier transform of the complex envelope, \( A = A(T, z) \), where \( \omega \) is radian frequency. In this thesis the Fourier transform used adheres to the physics convention, \( \tilde{A}(\omega, z) = \int_{-\infty}^{\infty} A(t, z) e^{j\omega t} dt \). (1.5) then becomes

\[
\frac{\partial \tilde{A}}{\partial z} = -\frac{\alpha}{2} \tilde{A} + \frac{i \beta_2}{2} \omega^2 \tilde{A}.
\] (1.6)

This differential equation is now readily solved [8, 17]

\[
\tilde{A}(\omega, z) = e^{\left(-\frac{\alpha}{2} z + \frac{i \beta_2}{2} \omega^2 z\right)} \tilde{A}(\omega, 0)
\] (1.7)

This can be thought of as an all-pass filter with nonlinear phase response. Were it not for the dispersion term, \( \frac{i \beta_2}{2} \omega^2 z \), the output would simply be an attenuated version of the input, scaled by the factor \( \exp\left(-\frac{\alpha}{2} z\right) \).

1.2.2 The Nonlinear Term

Ignoring the dispersion term, (1.4) becomes [8, 17]

\[
\frac{\partial A}{\partial z} = -\frac{\alpha}{2} A + i \gamma |A|^2 A
\] (1.8)

This differential equation can be solved approximately [8, 17]

\[
A(T, z) = e^{\frac{-\alpha}{2} z + i \gamma |A(T, 0)|^2 \frac{1 - e^{-\alpha z}}{\alpha}} A(T, 0)
\] (1.9)
Similar to the development in Section 1.2.1, if the nonlinear term were not present the output wave would simply be an attenuated version of the input. A brief examination of (1.9) might imply that the nonlinear effect only contributes a nonlinear phase shift. This would seem to imply that a modulation and detection scheme that is only a function of amplitude would render this phase distortion irrelevant. What is not being accounted for here is the effect of dispersion on the nonlinearity. If a fiber were completely free of dispersive effects the third-order nonlinearity outlined here would be harmless. It is precisely the interplay between dispersion and the nonlinearities that make this problem difficult to solve. Further insight into this interplay will be given in the description of the split-step Fourier method, in Section 3.2.

1.3 Fiber Dispersion

The focus of this thesis is not dispersive effects. However, to understand the impact of FWM and XPM a cursory description of dispersion must be given.

Dispersion is the mechanism responsible for the broadening of input pulses in the time domain. This can have severe impacts on system performance if individual pulses are broadened to the point of interfering with neighboring pulses.

There are three fundamental types of dispersion, modal, material and waveguide. In general modal dispersion is the most detrimental to system performance. Using Maxwell’s equations it can be shown that any given fiber geometry can support a finite number of hybrid and transverse modes [3]. This would not be a problem if it were not for the fact that each mode travels at a different speed through the fiber. Modal dispersion can be eliminated by using single mode fibers (SMF) [3]. This is done by simply extruding a fiber with a relatively small core radius. Single mode fibers are a popular choice in modern optical communication systems. The research presented in this thesis assumes a single mode fiber.
The other two types of dispersion, material and waveguide, constitute what is known as group-velocity dispersion (GVD). Due to GVD, different frequencies propagate at different speeds through the fiber. Because the spectrum of any pulse is nonsingular, some spectral components of the pulse travel faster than others. This results in a broadening of the pulse in the time domain. The dielectric material in the fiber core is responsible for this phenomenon [18].

The speed of propagation in a dielectric material is determined by the phase term $\beta$. In optical fibers this term is frequency dependent. To demonstrate how this frequency dependence causes dispersion consider a single spectral component transmitted at frequency $\omega$. This component would arrive at the output after a delay given by $T = L/v_g$ seconds, where $L$ is the length of the fiber and $v_g$ is the group velocity at the given frequency [3]. $v_g$ is defined as

$$v_g = \frac{1}{d\beta/d\omega}$$  \hspace{1cm} (1.10)

If $\beta$ were a constant, not dependent on frequency, then the phase shift would be constant and there would be no delay, let alone any dispersion. If $\beta$ were linearly dependent on frequency, the phase response would be linear, the group velocity would not be frequency dependent and there would still be no dispersion. The relationship between $\beta$ and frequency is more complicated however. Of particular interest is the second derivative, $\beta_2 = \frac{d^2\beta}{d\omega^2}$, which is known as the GVD parameter and has units $\frac{\text{ps}^2}{\text{km}}$. The dispersion parameter, $D_C$, can be expressed as [3]

$$D_C = \frac{d}{d\lambda} \left( \frac{1}{v_g} \right) = -\frac{2\pi c}{\lambda^2} \beta_2,$$  \hspace{1cm} (1.11)

From here the dispersion parameter can be split into material and waveguide components,

$$D_C = D_M + D_W,$$  \hspace{1cm} (1.12)
with material and waveguide components given as

\[
D_M = -\frac{2\pi}{\lambda^2} \frac{dn_{2g}}{d\omega} = \frac{1}{c} \frac{dn_{2g}}{d\lambda}, \quad \text{(1.13)}
\]

\[
D_W = -\frac{2\pi}{\lambda^2} \left[ \frac{n_{2g}^2}{n_{2g}} \frac{V d^2(Vb)}{dV^2} + \frac{dn_{2g}}{d\omega} \frac{d(Vb)}{dV} \right], \quad \text{(1.14)}
\]

where \( n_{2g} \) is the group index of refraction of the cladding material, \( \Delta \) is the index step, \( V \) is the normalized frequency and \( b \) is the normalized propagation constant.

From (1.13) and (1.14) it can be shown that \( D_W \) is negative for all \( \omega \), whereas \( D_M \) can be either positive or negative [2, 3]. The wavelength at which \( D_W = -D_M \) is referred to as the zero-dispersion wavelength, \( \lambda_{ZD} \). For standard single mode fibers \( \lambda_{ZD} = 1.31 \mu m \). Relatively high attenuation at 1.31\( \mu m \) lead to the development of dispersion-shifted fibers (DSF), which shift \( \lambda_{ZD} \) to 1.55\( \mu m \). Another recent development is dispersion-flattened fibers, which minimize dispersion throughout the 1.3 – 1.6\( \mu m \) low-attenuation window [3]. While dispersion-shifted fiber and dispersion flattened fiber have proven powerful in combating dispersive effects, an undesired side-effect is the amplification of non-linear effects such as four-wave mixing [8, 13, 19].

In addition to DSF, there are numerous other techniques that have been developed for eliminating group-velocity dispersion. Postcompensation systems are a relatively mature technology capable of substantially reducing dispersion. Historically this was done with electronic filters, but more recently optical dispersion compensation filters (DCF) have become popular. The philosophy is simply that if a fiber can be approximated as a linear system then dispersion can be filtered out with a response of the form [3]

\[
H(\omega) = e^{-\omega^2 \beta_2 L}. \quad \text{(1.15)}
\]

Another approach is referred to as dispersion management (DM). There exists a class of fibers for which the \( \beta_2 \) parameter can be either positive or negative. The theory is that if an optical communication system is made up of alternating fibers, \( \beta_2 = \pm B \), then dispersion can be eliminated incrementally throughout the system [3].
While GVD is the most destructive dispersive effect, there are others. There are higher order terms, $\beta_3$ and so on. Typically the terms $\beta_n$ for $n > 3$ are considered negligible, and often even $\beta_3$ is ignored as in this thesis. Another form of dispersion neglected so far is polarization dispersion. Any optical fiber supports two polarization modes. These two modes propagate with different group velocities due to birefringence. Birefringence is the term given to geometrical imperfections in the physical fiber. That is, the size and shape of any fiber is not perfectly uniform throughout the entire length of the fiber. While it is possible to input a wave such that it is contained entirely within only one polarization mode, as the signal encounters these imperfections energy is transferred from one polarization mode to the other. Ultimately this results in both modes carrying signal. A direct-detect photodetector does not resolve different polarizations modes, hence this is another source of dispersion [3].

1.4 Chapter Summary

This chapter delimits the basic problem of interest for this thesis. That is, in spite of the attention fiber optic nonlinearities have received, many fundamental questions remain about modulation formats that efficiently suppress these effects. A collection of different modulation formats are explored in scenarios where FWM and XPM are the limiting effects. In addition to presenting background and motivation this chapter introduces the nonlinear Schrödinger equation and a simple overview of chromatic dispersion. The next chapter goes into detail with the most important nonlinearities in fiber optic communication systems.
Chapter 2

Fiber Nonlinearities

2.1 Physical Description

Fused silica fibers are subject to a wide range of nonlinear effects [9]. In the early days of fiber optic communications these nonlinear effects were viewed as harmless novelties compared to the more serious problems of attenuation and dispersion. However, as data rates have steadily increased throughout the last two decades these same nonlinear effects have presented themselves as the primary limiting effects for future designs [8, 9]. There are five nonlinear effects in SMF DWDM systems, stimulated Raman scattering (SRS), stimulated Brillouin scattering (SBS), self-phase modulation (SPM), cross-phase modulation (XPM) and four-wave mixing (FWM).

An important figure of merit for these nonlinearities is the minimum input power per channel required for the nonlinear effect to be a problem. Figure 2.1, taken from [9], provides a comparison of this power threshold for four of the effects.

It may be tempting to make specific conclusions about which nonlinear effect is most severe from Figure 2.1. This figure should only be used for broad insight however, not specific conclusions. Figure 2.1 is useful for determining whether certain nonlinear effects can be neglected or not. There is still no absolute agreement on whether certain nonlinear effects are worse than others. This is due in part to the sheer complexity of the Generalized NLS equation (1.2), the dependence of the nonlinearities on a large number of system
parameters and the difficulty of separating the various nonlinear effects in experimental tests. It is well established that XPM and FWM are the most problematic nonlinear effects in modern WDM systems [8, 9, 10, 11]. Throughout the 1990’s FWM came to be viewed as the primary limiting factor by many researchers [9, 12, 13, 14, 15, 19, 20, 21, 22], although strong research has also been presented showing the effect of XPM to be stronger than FWM in many situations [11, 23, 24, 25].

![Graphs showing maximum power per channel](image)

(a) 10 GHz channel spacing  
(b) 1 GHz channel spacing

Figure 2.1: Maximum power per channel without substantial signal deterioration due to various nonlinear effects. Taken from [9]. Fig. (a) uses a channel spacing of 10 GHz and Fig. (b) uses a channel spacing of 1 Ghz. This figure assumes a DSF with center wavelength 1.55 $\mu$m, attenuation coefficient 0.2 $\text{dB/km}$, effective core area $5 \times 10^{-11} \text{m}^2$ and effective length 22 km.
The following subsections give a brief description of the fundamental physics behind each nonlinearity. A mathematical analysis of the third-order nonlinearities is given in the next section along with a simple two-channel example to illustrate the connection between the nonlinear Schrödinger equation presented in Section 1.2 with FWM and XPM. In the last section of this chapter some recently explored techniques for suppressing FWM are presented.

### 2.1.1 Stimulated Raman Scattering

Stimulated Raman scattering is an optical effect that can occur in any molecular medium. The general effect is to transfer a small fraction of optical energy, typically on the order of $10^{-6}$, down in frequency. The size of this frequency shift is determined by the vibrational modes of the medium. The frequency-shifted radiation is referred to as a Stokes wave [8].

For single channel designs SRS is not usually a problem. It has been shown that the effect of SRS is negligible below 500 mW total power [3]. Given that most fiber optic systems transmit less than 10 mW this might imply that SRS is not a problem. However, for WDM systems each channel will transmit up to 10 mW. Hence the total power of all the channels combined can easily exceed the 500 mW threshold [3].

Consider the two channel scenario. The effect of SRS is to transfer energy from the higher frequency channel to the lower frequency channel. For on-off modulation this energy transfer occurs only when both channels are transmitting a 1 bit, if either channel is transmitting a 0 bit there is no energy transfer. For channel spacing less than 15 THz the effect is proportional to channel spacing. That is, channels spaced farther apart are more effected by SRS than channels spaced close together [8, 9]. SRS clearly has a detrimental effect on the higher frequency channel. In general the lower frequency channel is also corrupted by SRS, but under some circumstances the effect of SRS on the lower channel can be beneficial [9].
In a WDM system the power penalty due to SRS can be calculated by [3]

\[
\delta_R = -10 \log_{10}(1 - D_{SRS}),
\]  

(2.1)

with parameter \( D_{SRS} \) given by

\[
D_{SRS} = \frac{\Delta F g_{R}^{\text{max}} P_{ch} L_{\text{eff}} N(N-1)}{6 \times 10^{13} A_{\text{eff}}}. 
\]  

(2.2)

In (2.2) \( \Delta F \) is channel spacing, \( g_{R}^{\text{max}} \approx 10^{-13} \text{m} \text{W} \) is the peak Raman gain, \( P_{ch} \) is the peak power per channel, \( N \) is the number of channels and \( L_{\text{eff}} \) is the effective fiber length given by \( L_{\text{eff}} = \frac{1 - e^{-\alpha L}}{\alpha} \).

A frequently used estimate is that the power penalty is less than 1 dB if the following relationship holds [9],

\[
NP_{ch} \Delta F(N - 1) < 500 \text{ GHz} \cdot \text{W}. 
\]  

(2.3)

From (2.3) it is observed that for a typical system with 50 GHz channel spacing and 10 mW peak power per channel, stimulated Raman scattering is not a problem so long as the total number of channels is below 30. The power limit imposed by SRS can be compared to that of the other nonlinearities in Figure 2.1. While SRS may prove to be a problem for future WDM systems with over 100 channels, other nonlinear problems are currently of more concern.

### 2.1.2 Stimulated Brillouin Scattering

Similar to SRS, stimulated Brillouin scattering is an effect that transfers signal energy from higher frequencies to lower frequencies. While superficially SRS and SBS might appear to be similar effects, there are several important differences. The physical mechanism responsible for SBS involves sound waves, whereas SRS involves molecular vibrations. The Stokes wave generated by SBS travels backwards relative to the incident wave. The Stokes wave generated by SRS travels in the same direction as the incident wave [8, 9].
The gain coefficient is two orders of magnitude larger for SBS than SRS. For a single channel system SBS can limit input power to 1 mW. Unlike SRS however, SBS has a very small gain bandwidth, 50 MHz. Because of this using wide-band pulses, over 500 MHz, can raise this threshold to 100 mW [3].

Because of this very narrow gain bandwidth, SBS does not directly cause interchannel crosstalk. The back-propagating Stokes wave generated by noise in WDM systems still limits input power however. Unlike SRS, this limit is not a function of the number of channels. The 1-dB power penalty threshold can be approximated using [3]

\[
\frac{g_B P_{ch} L_{\text{eff}}}{A_{\text{eff}}} < 21 \text{ W/m.}
\]

where \( g_B \approx 4 \times 10^{-11} \text{ m/W} \) is the Brillouin gain coefficient. As can be seen from Figure 2.1, stimulated Brillouin scattering is a concern for single channel systems but is quickly dwarfed by other nonlinearities in multi-channel systems. The effects of SRS and SBS are ignored in (1.4).

### 2.1.3 Carrier-Induced Phase Modulation

Self-phase modulation and cross-phase modulation, taken together, are referred to as carrier-induced phase modulation (CIP). SPM and XPM, as well as FWM, originate from nonlinear refraction, a phenomenon by which the refractive index of an optical fiber is dependent on the intensity of light passing through the fiber. This is known as the Kerr effect [8].

If \( n_1 \) and \( n_2 \) are the static linear refractive indices of the fiber core and cladding, respectively, then the effective nonlinear indices are given by [3]

\[
n'_1 = n_1 + \bar{n}_2 \left( \frac{P}{A_{\text{eff}}} \right)
\]

\[
n'_2 = n_2 + \bar{n}_2 \left( \frac{P}{A_{\text{eff}}} \right)
\]

where \( P \) is the total optical power in the fiber and \( \bar{n}_2 \) is the nonlinear-index coefficient, typically on the order of \( 3 \times 10^{-20} \text{ m}^2/\text{W} \) for silica fibers.
As the names imply, self-phase modulation is a single channel effect. Any channel being transmitted through optical fiber, either alone or amongst other channels, suffers from SPM. SPM is only a problem when a 1 bit is being transmitted, a 0 bit is unaffected by SPM [8].

On the other hand, XPM is a multi-channel effect, only systems transmitting multiple channels are affected. XPM is a two-channel effect in that any pair of channels produces its own XPM terms. XPM is only a problem when two 1 bits are transmitted simultaneously. That is, when two bits collide. If either channel is zero XPM does not occur [8].

The immediate net effect of SPM and XPM is a varying nonlinear phase shift on each channel. Ignoring the influence of GVD, the net phase shift for the $k^{th}$ channel is given by [3]

$$\phi_{NL}^k = \bar{\gamma} L_{eff} \left( P_k + 2 \sum_{l \neq k}^N P_l \right)$$

(2.7)

where $N$ is the number of channels and $P_k$ is the peak power of the $k^{th}$ channel. The modified nonlinear coefficient, $\bar{\gamma}$ is given by

$$\bar{\gamma} = \frac{k_0 \bar{n}_2}{A_{eff}}$$

(2.8)

where $k_0$ is Boltzmann’s constant. In (2.7) the first term in parentheses is the SPM contribution and the summation term is the XPM contribution. Notice the coefficient 2 in front of the XPM term, for a given channel the XPM induced by every other channel is said to be twice as efficient as the SPM induced by that channel. If all channels transmit with equal peak power $P_{ch}$ then (2.7) simplifies to

$$\phi_{NL}^k = \bar{\gamma} L_{eff} P_{ch} (2N - 1)$$

(2.9)

for all $k$. If SPM and XPM acted alone then the phase shift outlined above would be of no consequence to direct detection systems. However, due to the interaction between these nonlinearities and GVD, this phase shift is transformed into nonlinear amplitude modulations [3, 8].
Under certain circumstances, especially in single channel designs, the interaction between SPM and GVD can be useful. There exists a class of optical pulses, called fiber solitons, for which the effects of SPM and GVD cancel each other. Typically GVD results in a broadening of pulses in the time domain. However, there exists a class of chirped pulses such that GVD will result in a compression followed by an expansion of the pulse along the fiber. Generating this chirped pulse directly is difficult and unnecessary; the interaction between SPM and GVD induces a chirp in the pulse. Due to the intensity dependence of SPM it is possible to design a pulse such the proper chirp is induced and the pulse experiences no net compression or expansion during propagation [3].

Unfortunately, the effect of XPM cannot be canceled using solitons. From (2.7) and (2.9) it is easy to see the linear dependence on the number of channels. An estimate of the power threshold is given by [9]

$$ NP < 21 \text{ mW} \quad (2.10) $$

As can be seen from Figure 2.1, XPM is a limiting factor for many WDM scenarios.

2.1.4 Four Wave Mixing

Similar to XPM, four-wave mixing is a third-order nonlinear effect resulting from the dependence of the refractive index to the intensity of light. FWM can be viewed as a scattering process in which two photons of energies $\hbar \omega_1$ and $\hbar \omega_2$ create two new photons of energies $\hbar \omega_3$ and $\hbar \omega_4$ [3, 26].

Third-order nonlinearities are by no means limited to fiber optic systems. In theory any communications channel is subject to a countably infinite number of nonlinear effects, $2^{nd}$, $3^{rd}$, $4^{th}$ ... $n^{th}$ order and so on. Usually the even-order nonlinearities can be ignored because the frequency spacing between the transmitted channels and their even products is outside the bandwidth of interest. For optical fibers in particular these even-order nonlinearities simply do not occur because of symmetry in the silica material. In
spite of the fact that no practical nonlinear communications channel had yet been found, the potential effects of third and fifth-order nonlinearities on multi-channel communication systems were first postulated and documented in 1953 [27]. This is often cited as the original declaration of four-wave mixing, although at the time the term intermodulation interference was given to these effects. SPM, XPM and FWM are all examples of third-order nonlinearities. Silica fiber is subject to higher order nonlinearities, but usually these higher-order nonlinearities are negligible compared to FWM and XPM [8, 28].

Four-wave mixing in single-mode fibers was first documented in 1978 [26], although in 1978 the term three-wave mixing was used. Both naming conventions are reasonable. Three input waves are used to generate a fourth. The relationship between the frequencies of these waves is \(\omega_1 + \omega_2 = \omega_3 + \omega_4\), where \(\omega_1, \omega_2, \omega_3\) are the frequencies of the incident waves and \(\omega_4\) is the frequency of the generated wave. This relationship is usually expressed as \(\omega_4 = \omega_1 + \omega_2 - \omega_3\) by communication engineers [10, 12, 13].

The efficiency of FWM depends on what is referred to as the phase-matching condition, which is related to conservation of momentum [26]. This relates to the group velocities of the interacting waves. Due to GVD, different channels travel at different speeds, making FWM efficiency low. Modern fiber-optic systems operate near the zero-dispersion point however, meaning different channels travel at approximately the same speed. Hence FWM can be a problem. Often fiber optic systems are intentionally designed to leave some residual GVD so as to limit the efficiency of FWM [3, 8].

The distortion caused by FWM can be divided into two categories, spectral broadening and spectral intermodulation. Spectral broadening refers to the broadening of individual spectral lines whereas spectral intermodulation refers to the exchange of signal power between different spectral lines. The latter of the two distortions is more troublesome for WDM systems [26]. Using the continuous-wave assumption the power contained in a
spectral intermodulation component can be calculated by [12, 13, 10, 14, 26, 29]

\[ P_{klm}(L) = \eta \frac{D^2}{9} \gamma^2 P_k P_l P_m e^{-\alpha L} \left( \frac{(1 - e^{-\alpha L})^2}{\alpha^2} \right) \quad (2.11) \]

where \( \eta \) is the FWM efficiency, \( D \) is the degeneracy factor and \( P_k, P_l \) and \( P_m \) are the powers of the three incident channels. A more detailed analysis of this equation is given in Chapter 4.

An estimate of the FWM power threshold can be seen in Figure 2.1.

### 2.2 Mathematical Description

Perhaps the most useful understanding of SPM, XPM and FWM can be had by considering how these effects are modeled in the nonlinear Schrödinger equation (1.4). The third-order nonlinear term of the NLS equation is

\[ \gamma |A|^2 A. \quad (2.12) \]

Consider the two-channel scenario and ignore the distance dependance for the moment.

Let

\[ A(t) = A_1(t) + A_2(t) = B_1(t)e^{i\omega_1 t} + B_2(t)e^{i\omega_2 t} \quad (2.13) \]

where \( B_1 \) and \( B_2 \) are the complex envelopes of each individual channel and \( \omega_1 \) and \( \omega_2 \) are the frequencies of each channel. Then the expanded nonlinear term is [8, 9, 17],

\[ |A|^2 A = A_1 A_1^* A_1 + A_2 A_2^* A_2 + A_1 A_1^* A_2 + A_2 A_1^* A_1 + A_1 A_2^* A_1 + A_2 A_1^* A_2 \]

\[ = \left| B_1 \right|^2 B_1 e^{i\omega_1 t} + \left| B_2 \right|^2 B_2 e^{i\omega_2 t} + \left| B_1 \right|^2 B_2 e^{i\omega_2 t} + \left| B_2 \right|^2 B_1 e^{i\omega_1 t} + B_1^2 B_2 e^{i(2\omega_2 - \omega_1) t} + B_2^2 B_1 e^{i(2\omega_1 - \omega_2) t}. \quad (2.14) \]
In the equation above the separate components due to SPM, XPM and FWM are easy to see. The $|B_1|^2 B_1 e^{i\omega_1 t}$ and $|B_2|^2 B_2 e^{i\omega_2 t}$ terms represent self-phase modulation. The two $|B_1|^2 B_2 e^{i\omega_2 t}$ terms and two $|B_2|^2 B_1 e^{i\omega_1 t}$ terms represent cross-phase modulation. And lastly, the $B_1^* B_2^* e^{i(2\omega_1 - \omega_2) t}$ and $B_2^* B_1^* e^{i(2\omega_2 - \omega_1) t}$ terms represent four-wave mixing.

Each channel distorts itself through SPM. Additionally, each channel suffers twice as much distortion from every other channel through XPM. In this particular case the two FWM terms do not interfere with the incident channels. However, signal energy is still depleted from the original channels to generate these new extraneous channels located at frequencies $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$.

This is for a simple two-channel case. The situation becomes much more complicated as additional channels are added. For an $N$ channel system, the $\gamma |A|^2 A$ nonlinearity will generate $N^3$ terms, some of which fall on existing channels and some of which do not. If the channels are equally spaced a total of $\frac{1}{12} (4N^3 - 9N^2 + 2N + 6)$ components will interfere with the incident channels [12, 27]. $N$ self-phase modulation terms, $2N(N - 1)$ cross-phase modulation terms and $N^3 - 2N^2 + N$ four-wave mixing terms are generated.

If the channels are labeled 1 through $N$, then new signal components are placed at frequencies,

$$\omega_m = \omega_k + \omega_l - \omega_m \quad (2.15)$$

for all $1 \leq k, l, m \leq N$. If $k = l = m$, then the component is SPM. If $k = m$ or $l = m$, but $k \neq l$, then the component is XPM. All other components, that is $m \neq k, l$, are FWM. If $k = l$ the component is degenerate FWM, if $k \neq l$ the component is nondegenerate FWM.

While FWM generates $N^3 - 2N^2 + N$ mixing products, not all of these products are at unique frequencies. In fact, only $\frac{N^2}{2} (N - 1)$ new frequencies are generated [12]. This is because for a given $K \neq L \neq M$ there are two mixing products that transfer energy to the same frequency, $\omega_K + \omega_L - \omega_M$ and $\omega_L + \omega_K - \omega_M$. Whereas for $K = L \neq M$ there is
only one mixing product, \( \omega_K + \omega_K - \omega_M \). Degenerate FWM refers to frequencies subject to only one mixing product, nondegenerate FWM frequencies are subject to two mixing products.

Consider a two and three channel continuous wave spectrum. The four-wave mixing components can be seen in Figure 2.2 [9]. The channel labels are \( F_1, F_2 \) and \( F_3 \), the FWM products are labeled \( f_{klm} \) where \( k, l \) and \( m \) specify which channels are being mixed. Notice, in the three channel spectrum, if the three channels had been equally spaced then some of the FWM components would have intersected with the information channels.

\[ \text{Figure 2.2: Four-wave mixing products for a two and three channel CW spectrum} \]

2.3 FWM Suppression

Numerous techniques have been proposed during the past decade with the hope of softening the effect of third-order nonlinearities on WDM systems. Most of these techniques have focused on FWM, although some of these techniques apply equally well for suppressing XPM. A few common themes run between these techniques. Most of them either try to minimize the number of FWM terms that intersect with information channels, minimize the phase-matching coefficient, minimize the number of "collisions" throughout the fiber or suppress the carrier. To date, no approach has found broad acceptance. Each technique demonstrates potential under certain circumstances, but also has drawbacks.
It should be noted that all of the techniques described below address FWM only in the WDM scenario, not the DWDM scenario. That is, while numerous channels are being considered, the channel spacing is relatively large, ranging from 50 to 500 GHz. This is somewhat different to the situation considered in this thesis, where the channels are densely packed with 25 GHz channel spacing. While the techniques presented here are valid for large channel spacing their efficacy with small channel spacing is questionable.

2.3.1 Unequal Channel Spacing

In the middle 1990s the technique of using unequal channel spacing to suppress FWM was proposed [30, 31]. The idea has subsequently been studied extensively and expanded [6, 19, 32, 33, 34]. This is one of the most fully developed approaches to FWM suppression. Because unequal channel spacing relates directly to a technique explored with this project, a thorough description is saved for Chapter 5.

An idea closely related to unequal channel spacing is channel detuning. One of the immediate complications of unequal channel spacing is that current device technology restricts WDM channels to be equally spaced. This is written into the formal ITU standard. Channel detuning refers to the small random frequency shift applied to each channel, on the order of 5 GHz. That is, the channels are still essentially equally placed, but due to the modest detuning fewer FWM mixing terms interfere with the spectral lines in the information channels. For systems with channel spacing on the order of 200 Ghz, with data rates of 10 Gb/s per channel, channel detuning has been shown to be useful in diminishing the effect of FWM [35].

2.3.2 Using GVD to suppress FWM

The most common approach to minimizing FWM is to not eliminate GVD entirely. Prior to the invention of dispersion-shifted fibers and dispersion-flattened fibers chromatic dispersion had been the limiting factor. Thanks to these fibers GVD has been substantially di-
minished. However, minimizing GVD maximizes the phase-matching condition that FWM depends on. A simple solution to this is to allow some modest GVD to remain, this will prevent FWM from being a major problem. Typically this is done by offsetting the center wavelength of transmission from the zero-dispersion wavelength. The obvious drawback to this approach is that FWM, while not catastrophic, still limits system performance and degradation due to GVD is worse than it otherwise would have been [3, 8].

Further improvement can be had by using a system with nonuniform chromatic dispersion. This can take the form of either a system in which multiple concatenated fibers are used with each fiber having a slightly different zero-dispersion wavelength, or a system in which the dispersion varies continuously throughout the length of each individual fiber. Both approaches have been studied with the intention of alleviating dispersive effects, this is called dispersion management [3, 8]. The use of dispersion management specifically to address FWM is a relatively recent idea.

The use of dispersion-shifted fibers with alternating zero-dispersion wavelengths was explored in [14]. This paper demonstrated that for reasonably large channel spacing, 100 GHz, FWM in a continuous-wave system could be substantially reduced using alternating fibers. However, it was also shown that this technique fails to show any improvement when the channel spacing becomes small, 10 GHz. Later, the use of dispersion-varying fibers (DVF) was explored [36]. For a channel-spacing of 200 GHz and a data rate of 2.5 Gb/s this technique was very effective at eliminating FWM. Denser channel spacing configurations were not considered.

2.3.3 Other Techniques

A hybrid WDM/TDM system was proposed in [37, 38]. The work compared a twelve channel WDM system to hybrid system. The hybrid system multiplexed three sets of four channels using TDM and transmitted these three channels using WDM. This corresponds
to increasing the channel spacing from 100 GHz to 400 GHz and increasing the data rate from 2.5 Gb/s to 10 Gb/s. Performance of the hybrid system was found to be significantly better than the conventional WDM system. While this technique does suppress FWM, the system is also rendered more vulnerable to dispersion and other device limitations, such as nonideal photodetector response. [39] explored a similar question and found the hybrid system to yield no performance improvement due to exacerbated dispersive effects.

As mentioned in Section 2.1.4, four-wave mixing products are only generated when, at a given point along the fiber path, multiple channels simultaneously carry signal energy. The technique of bit-arranged return-to-zero (BARZ) pulse shaping has been explored in [40, 41, 42]. BARZ pulse shaping attempts to minimize the number of collisions as the channels propagate through the fiber. By using very short pulses FWM was minimized in analytic and simulated results. The success of this technique is limited because ultra-short pulses introduce higher order dispersive and nonlinear effects.

Recently the use of RZ pulse shaping coupled with carrier-suppressed modulation has shown potential for some WDM configurations [38, 43, 44]. The large spectral line that results from standard amplitude modulation contributes significantly to the effect of XPM and FWM. By not explicitly transmitting the carrier there is less nonlinear interference. To date the only work on the subject has been experimental, focusing on the 40 Gb/s per channel, 100 GHz channel spacing scenario. Little work has been done comparing this method directly to other methods.

Wavelength Shift Keying (WSK) has also been shown to be effective in mitigating FWM in WDM systems [45]. Due to the dependence of the phase-matching coefficient on GVD, FWM efficiency is symmetric with respect to the zero-dispersion wavelength. By transmitting each signal using two channels on either side of the zero-dispersion wavelength, one channel for a 0 bit and the other for a 1 bit, the impact of FWM components falling on any single channel is half that of a standard on-off keying system. This tech-
nique does not make efficient use of available bandwidth and requires a more complicated balanced receiver however.

Another technique is to take advantage of the fact that single-mode fibers support two polarization modes. If adjacent channels use orthogonal polarization modes there is no FWM interaction between them. However, due to birefringence it is impossible to keep the polarization modes orthogonal throughout the entire length of the fiber [8].

In analog broadcast WDM systems the use of complementary twin carriers has been recently proposed [46]. While this is a somewhat different application than high-capacity digital links, the results are intriguing. Optical CATV systems broadcast multiple analog RF signals in the 50 to 1500 MHz bandwidth. By transmitting for each channel two complementary carriers at very close frequencies the overall envelope for each channel is constant. This reduces the effect of XPM and FWM to a constant offset. The term walk-off is used to describe the phenomenon which causes different channels to propagate at slightly different speed. The walk-off effect between the two carriers due to dispersion is minimal for CATV applications, however for high-capacity digital links this effect is substantial and the constant envelope condition will break.

2.4 Chapter Summary

This chapter presents the nonlinear effects of interest to contemporary fiber optic communication systems. As described above, of particular interest are cross-phase modulation and four-wave mixing. Numerous techniques have been proposed to alleviate the effects of XPM and FWM, but have proven useful only in limited circumstances. The following chapter presents the simulation model used in this research and describes the algorithm used to simulate these nonlinear effects.
Chapter 3

System Description

3.1 System Model

A block diagram for the system being simulated is given in Figure 3.1. This is the fundamental structure for a single-span intensity-modulated direct-detect (IM-DD) WDM system. Optical device technology is a rich and fast paced field of study, it is beyond the scope of this work to pay particular attention to the current state-of-the-art in device technology. The necessary approximations are made to model contemporary systems without attempting to capture every minute device limitation. To simplify computation, simulations are run at baseband rather than optical frequencies. In general, nonlinear systems cannot be accurately modeled using baseband. However, the nonlinearities of interest here are intensity dependent, not frequency dependent. Hence a baseband model is valid.

Figure 3.1: System Model
The system being modeled is noiseless. This is clearly an unrealistic assumption. Nonetheless, the results found are still legitimate. Noise enters a WDM system at virtually every step in the system. Amplified spontaneous-emission (ASE) noise from the optical amplifiers, shot noise from the photodetectors and thermal noise from the electrical system are often seen as the most troublesome noise sources. Ironically, the optical fiber itself is relatively free of noise [3]. Noise in nonlinear optical systems is difficult analyze; a Gaussian noise source is no longer Gaussian after passing through a nonlinear fiber [8]. None of the techniques explored with this research hope to alleviate the effects of noise though. As far as this project is concerned noise is a continuous effect that disturbs all configurations equally. This assumption is frequently used in the study of nonlinear fiber optics [6, 9, 10, 12, 13, 14, 15, 25, 30, 31, 32, 33, 36, 37, 46, 47].

3.1.1 Optical Sources

The optical source is frequently a semiconductor laser, although light-emitting diodes are also used. Each of the $N$ lasers transmits with a unique carrier frequency. WDM systems use either an array of discrete lasers or, more commonly, a single monolithic transmitter with numerous lasers. Both configurations require some sort of tuning device to set the carrier frequency for each channel. This is typically done with a Braggs grating, which only reflects light within a narrow bandwidth, and the rest is absorbed. In the simulations here each source is assumed to emit light at a single frequency [3].

There are two approaches to modulating information onto an optical signal. Classical designs use a constant light source that is externally modulated. More contemporary designs simply modulate the electrical current supplied to the laser, which in turn modulates the intensity of the light output. The intensity of the light output is proportional to the current input. This so called intensity-modulated (IM) system is used here. Gaussian pulses are used throughout this research, although super-Gaussian and raised cosine pulses are
also popular choices. The Gaussian class of pulses are of the form [3],

\[ P(t) = P_{ch}e^{-\frac{t^2}{2T_0^2}}, \tag{3.1} \]

where \( P_{ch} \) is the transmitted peak power, \( T_0 \) is the pulse width and \( m = 1 \) for a Gaussian pulse and \( m > 1 \) for a super-Gaussian pulse. As \( m \) increases the pulse becomes more rectangular.

Both non-return-to-zero (NRZ) and return-to-zero (RZ) pulse shaping is considered in this thesis. Throughout the last half decade there has been an active debate concerning NRZ versus RZ pulse shaping in WDM systems [25, 47, 48, 49, 50, 51, 52, 53, 54]. There are no broadly applicable guidelines for determining whether NRZ or RZ pulse shaping is preferable. In this thesis NRZ and RZ pulse shapes are compared for various scenarios, however general conclusions are not drawn from the observations.

### 3.1.2 WDM Multiplexer

WDM multiplexers and demultiplexers are the same physical device, the only difference being what is labeled input and output. Multiplexers take multiple light sources and combine them into a composite optical signal. Demultiplexers split this composite signal into its respective frequency components or channels. Both devices rely on a frequency selective component. This can either be a diffraction grating and a lens or a collection of narrow filters and directional couplers [3].

If all the light sources are sufficiently narrowband the multiplexer is adequately modeled as a linear summing device, as is done in this thesis. The demultiplexer can be modeled as an array of low-order filters. For this thesis third-order butterworth filters are used. For this project the multiplexers and demultiplexers are assumed to be completely flexible, that is, capable of handling any distribution of channel frequencies. This is a somewhat unrealistic assumption because all existing multiplexers and demultiplexers require uniform channel spacing. It is not required that all channels be used however. For example, to
implement a three channel system where channels 1 and 2 are separated by 25 GHz and channels 2 and 3 are separated by 50 GHz, a 4 channel multiplexer could be used in which all channels are uniformly spaced at 25 GHz and one of the center channels is not used.

### 3.1.3 Optical Fiber

A single span of step-index single-mode fiber is simulated. Step-index single-mode fibers are standard for long-haul communications. The length simulated is 50 km and the nonlinear coefficient, \( \gamma \), is \( 2 \frac{1}{W \cdot \text{km}} \). Due to the complexity of the NLS equation, presented in Section 1.2, there is no closed form expression for the optical wave at the end of a fiber. Several numerical techniques exist, with the split-step Fourier (SSF) method being the most popular for simulating the effects of SPM, XPM and FWM. This is the method used in this research. The SSF algorithm is described in the Section 3.2. The effects of SRS and SBS are negligible in the scenarios explored here [8].

Dispersion-shifted fiber is simulated. The center wavelength is the standard 1.55 µm. This places the signals in the center of the low-loss region of the fiber. Fiber attenuation is, in general, frequency dependent. Within the low-loss region it is sufficient to model the attenuation as constant, \( \alpha \).

As mentioned in Section 1.3, chromatic dispersion in optical fibers is a result of the frequency dependence of the phase term \( \beta \). The GVD parameter is given as \( \beta_2 = \frac{d^2 \beta}{d\omega^2} \). In theory \( \beta_n = \frac{d^n \beta}{d\omega^n} \neq 0 \), for all integer \( n > 0 \). The effect of \( \beta_{n>2} \) is negligible compared to that of \( \beta_2 \) when individual channel rates are less than 40 Gb/s, as is the case in this work. For the region of interest in this thesis \( \beta_2 \) is assumed to be independent of frequency, that is \( \beta_{n>2} = 0 \) [3].

Recall that transmitting at precisely the zero-dispersion wavelength results in catastrophic four-wave mixing [8]. For this reason the center frequency is offset from the zero-dispersion wavelength, with \( 1 < \beta_2 < 2 \frac{\text{ps}^2}{\text{km}} \). This is the typical low-dispersion domain for WDM sys-
dispensation systems come in several different forms. One common type is to use a "short" fiber with a dispersion of opposite sign. For example, the dispersion incurred through 50 km of fiber with $\beta_2 = 2 \text{ ps}^2 / \text{km}$ could be balanced using 10 km of fiber with $\beta_2 = -10 \text{ ps}^2 / \text{km}$. This is referred to as a dispersion compensation fiber (DCF). Other methods use filters and gratings so that different frequency components travel slightly different lengths, resulting in all frequency components realigning. These optical dispersion compensation techniques have proven effective for reasonably large bandwidths. In general these devices also demonstrate their own nonlinear effects which corrupt the signal. These effects are entirely dependent on specific device characteristics and are small compared to the nonlinearities in the long-haul fiber. In this thesis the dispersion compensator is assumed to be perfect, that is, able to perfectly realign the different frequency components without inducing additional nonlinearities [17].

Erbium-doped fiber amplifiers (EDFA) have become the amplifier of choice for WDM systems. The bandwidth of previous WDM systems had been limited not by the fiber, but the amplifier. Current EDFAs have a bandwidth on the order of 1 THz [3]. While the bandwidth is very wide, the amplitude response is not completely flat and the phase response is not completely linear. These are well known deterministic effects however, and hence are easily accounted for. In this thesis the amplifier response is assumed to be perfect normalization.
3.1.5 Optical Receivers

After the composite signal has been separated into channels a photodiode is used to direct detect (DD). This is a non-coherent receiver. The current output is the power envelope of the optical signal, with no dependence on the phase of the optical signal. In general the impulse response of a photodiode is not an ideal impulse. However for reasonably long pulse widths, greater than 20 ps, the response can be approximated with an ideal impulse.

Although the demultiplexer partially filters neighboring channels, further electronic filtering is needed to fully isolate the channel. Being back in the electrical domain there is a wide variety of filters available. In this thesis an ideal rectangular filter is used.

The electronic signal is then sampled to restore the original digital data. Typical binary IM-DD systems with on-off keying use a simple threshold rule to make bit decisions. If, at the sampling instant, the signal power is over some threshold a $1$ is detected, otherwise a $0$ is detected. More sophisticated detectors, such as integrate-and-dump detectors, are not yet capable of operating at optical system speeds.

3.1.6 Measuring System Performance

There is no generally accepted method for measuring fiber optic system performance. Many methods are quite similar, but no single method has found universal acceptance.

The most obvious method of measuring communication system performance, bit-error rate (BER), is impractical for numerical simulations of optical systems. Optical communication systems are typically expected to operate with a BER on the order of $10^{-9}$. Simulating enough iterations to accurately measure such a low BER is not an efficient use of computing power. Deriving analytic expressions for BER has also proven difficult except in limited circumstances because there is no general solution to the NLS equation (1.4) and the nonlinear dependence of system behavior on a large number of parameters. Experimental systems will often report performance in BER however.
A closely related measure of system performance is the $Q$ factor. Under the Gaussian noise and interference model, the $Q$ factor is defined as [3]

$$BER = \frac{1}{2} \text{erfc} \left( \frac{Q}{\sqrt{2}} \right) \approx \frac{e^{Q^2}}{Q\sqrt{2\pi}}.$$  \hspace{1cm} (3.2)

The $Q$ factor for binary on-off keying can be obtained from the sample mean and variance by [3]

$$Q = \frac{\mu_1 + \mu_0}{\sigma_1 + \sigma_0},$$  \hspace{1cm} (3.3)

where $\mu_1$ and $\sigma_1$ are the measured mean and variance of the 1 bit and similarly for the 0 bit. The sample variances, $\sigma_1$ and $\sigma_0$, are not just the result of noise; these terms also account for nonlinear interference.

Receiver sensitivity is a popular choice for numerical simulations. Receiver sensitivity is defined as the minimum power required at the receiver to keep the ultimate BER above some arbitrary threshold, often $10^{-6}$ or $10^{-9}$.

Often the dependence on BER is discarded entirely. Measuring either the minimum eye-size directly or calculating the eye-closure penalty is common [12, 20, 30, 31, 33, 37, 39, 47, 48, 52, 53, 54]. The eye-closure penalty is a measure of the minimum received eye-size relative to input eye-size scaled by the attenuation factor.

This thesis uses both the minimum eye-size and the eye-closure penalty. The philosophy is that if a communication system is limited by noise then the minimum eye size is the relevant figure of merit, whereas if the system is limited by interference then the eye-closure penalty is more important. These measurements can be related to the $Q$ factor described above, except the variance terms are the result of nonlinear interference and not noise. Part of this thesis explores multilevel signaling, where transmitted pulses can take on more than two levels. There is no generally accepted method for defining the $Q$ factor with multilevel signals and hence $Q$ factor is not considered here.

Sometimes no measure of system performance is taken. This is frequently the case for research concerning strictly fiber nonlinearities [9, 10, 13, 23, 26, 29, 32]. These works
typically focus on the power of channel interference due to FWM and XPM. Some of the analytic work presented in this thesis uses this measurement to report these effects.

### 3.2 Split-Step Fourier Method

As shown in Section 1.2, if the dispersion and nonlinear terms of the NLS equation can be separated then an approximate closed-form solution can be found. The philosophy behind the split-step Fourier (SSF) method is that for a short segment of fiber, $h$, it is valid to assume the two effects act independently. The SSF method divides the entire fiber length into a sequence of adjacent segments and processes each segment in turn. This method is frequently used to model nonlinear fiber systems [8, 12, 19, 30, 33, 37, 42, 48, 49, 52]. The NLS equation can be rewritten as [8]

$$\frac{\partial A}{\partial z} = (\hat{D} + \hat{N})A, \quad (3.4)$$

where the operators $\hat{D}$ and $\hat{N}$ are defined as

$$\hat{D} = -i\beta_2 \frac{\partial^2}{\partial T^2} - \frac{\alpha}{2}, \quad (3.5)$$

$$\hat{N} = i\gamma |A|^2. \quad (3.6)$$

If the operator $\hat{N}$ is taken to be $z$ independent, then an exact solution is given by [8]

$$A(T, z + h) = e^{h(\hat{D} + \hat{N})} A(T, z). \quad (3.7)$$

For a sufficiently short segment, $h$, the operators $\hat{D}$ and $\hat{N}$ can be assumed commutative. Hence it is possible to first compute the contribution due to the nonlinear operator, setting $\hat{D} = 0$, and then the contribution due to the dispersion operator, $\hat{N} = 0$ [8],

$$A(T, z + h) \approx e^{h\hat{D}} e^{h\hat{N}} A(T, z). \quad (3.8)$$

The very first assumption, that $\hat{N}$ is independent of $z$, is of course false. Removing this assumption, while still assuming that $\hat{D}$ and $\hat{N}$ commute yields

$$A(T, z + h) \approx \exp(\hbar \hat{D}) \exp \left( \int_{z}^{z+h} \hat{N}(z')dz' \right) A(T, z). \quad (3.9)$$
In Section 1.2.1 it is shown that the Fourier transform is efficient for solving the dispersion portion of the NLS equation. Therefore (3.8) becomes

$$A(T, z + h) \approx \mathcal{F}^{-1}\left\{ e^{h\hat{D}(\omega)} \mathcal{F}\left\{ e^{h\hat{N}} A(T, z) \right\} \right\},$$

(3.10)

where $\mathcal{F}$ and $\mathcal{F}^{-1}$ represent the Fourier transform and inverse Fourier transform, respectively, and $\hat{D}(\omega)$ is the Fourier transform of the dispersion operator, $\hat{D}$. Central to the relative speed of the split-step Fourier method is the availability of the fast-Fourier transform (FFT). Without the FFT and inverse FFT the split-step Fourier method would be no faster than other methods.

The integration in (3.9) can be more accurately approximated using the trapezoidal rule instead of simply $h\hat{N}$ [8]

$$\int_{z}^{z+h} \hat{N}(z')dz' \approx \frac{h}{2} \left( \hat{N}(z) + \hat{N}(z+h) \right).$$

(3.11)

Implementation is complicated by the $\hat{N}(z+h)$ term, which is not known until after computing $A(T, z + h)$. An iterative process can be used to resolve this difficulty. The first iteration assumes $\hat{N}(z+h) = \hat{N}(z)$, which is used to calculate an initial estimate of $A(T, z + h)$. This is then used to refine the estimate of $\hat{N}(z+h)$. Typically two or three iterations are sufficient [8].

The nonlinear effect is distributed continuously throughout the length of the segment. The split-step Fourier method developed thus far lumps the nonlinear effect into one discrete point at the beginning of each segment however. It has been shown [8] that improved accuracy results from lumping the nonlinear effect into the middle of each segment. This is referred to as the symmetrized split-step Fourier method. Mathematically this results in rewriting (3.8) as

$$A(T, z + h) \approx e^{\frac{h}{2}\hat{D}} e^{h\hat{N}} e^{\frac{h}{2}\hat{D}} A(T, z).$$

(3.12)

Taking into account each development described above, one complete iteration of the algo-
The algorithm is given by

\[
A(T, z + h) \approx \mathcal{F}^{-1} \left\{ e^{\frac{h}{2} \tilde{D}(\omega)} \mathcal{F} \left\{ e^{\frac{h}{2} (\hat{N}(z) + \hat{N}(z+h))} \mathcal{F}^{-1} \left\{ e^{\frac{h}{2} \tilde{D}(\omega)} \mathcal{F} \{ A(T, z) \} \right\} \right\} \right\},
\]

where \( \hat{N}(z + h) = \hat{N}(z) \) for the first iteration.

Choosing an appropriate segment length, \( h \), is crucial to an efficient and accurate implementation of the SSF method. The immediate effect of the third-order nonlinearities is a phase shift. The maximum phase shift that could be induced in a segment of fiber with length \( h \) is given by \( \phi = \gamma P_{\text{max}} h \), where \( P_{\text{max}} \) is the maximum total power that segment of fiber is subject to. The accuracy of the SSF method is maintained by restricting the allowable phase shift below some arbitrary value, that is, \( \gamma P_{\text{max}} h \leq \phi_{\text{max}} \) for every segment. For small \( \phi_{\text{max}} \) the algorithm is accurate but slow. For this research a \( \phi_{\text{max}} = 0.005 \) radians is used.

The speed of the algorithm is improved by calculating a new segment length, \( h \), for every segment. A wave is naturally attenuated as it propagates through a fiber, and hence the allowable \( h \) can increase slightly with each segment.

Figure 3.2, based on a similar Figure in [8], provides an illustration of the split-step Fourier method used in this thesis.

![Figure 3.2: Illustration of symmetrized split-step Fourier method. Notice the varying segment widths, \( h_n \).]
3.3 Chapter Summary

This chapter presents the system being simulated in this research. The fundamental algorithm used to model the third-order nonlinearities, the split-step Fourier method, is introduced. Using this algorithm several modulation techniques are explored in the following chapters.
Chapter 4

Multilevel Signaling

4.1 Motivation

Multilevel signaling in fiber optic communication systems is a relatively unexplored area. Throughout the 1990’s scattered research was done in multilevel signaling in fiber, however the focus at the time was mitigating dispersive effects [55, 56, 57, 58]. Recently the use of duobinary modulation was presented in the context of a nonlinear crosstalk limited system [44], but no comparison with other modulation formats was made.

Chromatic dispersion is a phenomenon by which different frequency components travel at different velocities. A signal with larger bandwidth is subject to more chromatic dispersion than a signal with smaller bandwidth. A multilevel signal carrying the same data rate as a standard binary signal occupies less bandwidth and hence is subject to less dispersion.

This bandwidth compression is of no immediate benefit when attempting to alleviate nonlinear effects however. By compressing the bandwidth of a signal, while maintaining the same average power, more power is concentrated around the carrier. Four-wave mixing and cross-phase modulation are power dependent nonlinearities [8]. Hence, the use of multilevel signaling will decrease the effect of nonlinearities in the side-lobes but increase the effect close to the carrier. It is not immediately obvious whether using a multilevel signaling format in place of a conventional binary format will yield any performance improvement.

However, it is well known that dense channel spacing exacerbates the effects of four-
wave mixing [8, 9, 12, 29]. The question of using multilevel signaling to increase channel spacing while maintaining the same bandwidth efficiency (in \( \text{bits/s} \cdot \text{Hz} \)) as conventional systems has not been explored. This question is addressed in this chapter.

A related question has been addressed in previous research. Within the domain of conventional binary signaling formats with a fixed bandwidth efficiency, the question of what is the optimal data rate per channel and channel spacing has been explored [8, 37, 38, 39]. For example, does transmitting 16 channels at 10 Gb/s with 25 GHz channel spacing yield better performance than transmitting 4 channels at 40 Gb/s with 100 GHz channel spacing? In general there is no simple answer to this question. [39] showed that the two scenarios outlined above yield essentially identical performance in conventional fiber systems, the former limited by nonlinearities and the latter by dispersion. On the other hand [37] showed that the latter scenario yields a performance improvement if the system is operating at the zero-dispersion point. The use of such fibers is not typical however because FWM and XPM are catastrophic with very low-dispersion.

Using multilevel signaling it is possible to increase the data rate per channel, and hence reduce the number of channels and increase the channel spacing, while still maintaining roughly the same bandwidth per channel. That is, this technique could alleviate third-order nonlinearities while not making the system any more susceptible to dispersion. This could improve system performance. Analytic and numeric simulations are presented in this chapter to demonstrate the potential improvement under certain circumstances.

### 4.2 Basic Concepts

Two multilevel modulation formats are explored, polybinary and \( M \)-ary ASK. Comparing the bandwidth of these two formats with that of regular binary signaling is useful. Assuming a normalized rectangular non-return-to-zero (NRZ) pulse shape and neglecting the spectral line terms, the power spectral density (PSD) of a conventional binary signal is
CHAPTER 4. MULTILEVEL SIGNALING

given by [59, 60],

\[ G_{\text{binary}}(\omega) = T_b \text{sinc}^2(T_b\omega), \quad (4.1) \]

where \( T_b \) is the bit period and \( \text{sinc}(\Omega) = \frac{\sin(\Omega)}{\Omega} \).

Polybinary signaling was first described in the 1960’s [60], at the time referred to as cor-
relative level coding for binary signals. The philosophy behind polybinary signals is that by
introducing correlation between adjacent bits it is possible to concentrate the PSD around
the carrier. This is done at the cost of introducing multiple levels into the signal transmitted.
Regardless of the number of levels, each polybinary symbol transmitted carries one bit of
information. For a polybinary system with \( b \) levels, the current symbol transmitted depends
on the current information bit and the \( b - 2 \) previous information bits [55, 60].

Given the original information bits, \( \{d_k\} \), generating the polybinary sequence requires
two steps. First a temporary sequence, \( \{t_k\} \), is made according to [55],

\[ t_k = d_k \oplus t_{k-1} \oplus t_{k-2} \cdots \oplus t_{k-(b-2)}, \quad (4.2) \]

where \( \oplus \) represents addition modulo two. The polybinary sequence, \( \{p_k\} \), is then computed
by,

\[ p_k = t_k + t_{k-1} + \cdots + t_{k-(b-2)}. \quad (4.3) \]

Note that the original data sequence is related to the polybinary sequence by,

\[ d_k = p_k \mod 2. \quad (4.4) \]

With the same assumptions as in (4.1), the PSD for a polybinary signal with \( b \) levels is
given by [55, 60],

\[ G_{\text{polybinary}}(\omega) = \frac{(b-1)^2 T_b}{2} \text{sinc}^2((b-1)T_b\omega). \quad (4.5) \]

Notice that the bandwidth is compressed by a factor of \( b - 1 \) compared to the conventional
binary case. Of interest in this work is the so called duobinary case, \( b = 3 \). The bandwidth
of a duobinary signal is half that of the corresponding binary signal. In [55] it was shown that further increasing \( b \) does not improve performance.

\( M \)-ary amplitude shift keying (ASK) is a classic digital communication format. An \( M \)-ary ASK symbol can take on one of \( M \) levels. Each symbol then represents \( \log_2 M \) information bits. The symbol period \( T_s \) is related to the bit period \( T_b \) by \( T_s = \log_2 M \cdot T_b \).

With the same assumptions as in (4.1), the PSD for an \( M \)-ary signal is given by [59],

\[
G_{4-ary}(\omega) = T_s \sin^2(\omega T_s),
\]

(4.6)

Similar to the polybinary PSD, the bandwidth of an \( M \)-ary signal is compressed by a factor of \( \frac{T_s}{T_b} \) relative to the binary case. This research focuses on the 4-ary case in which the bandwidth is compressed by a factor of two.

Hence a binary signal operating at 10 Gb/s, a duobinary signal operating at 20 Gb/s and a 4-ary signal operating at 20 Gb/s all occupy the same bandwidth.

### 4.3 Configurations

The scenarios under consideration here are intentionally chosen to be limited by four-wave mixing. That is, only long-haul scenarios are considered. As outlined in the first chapter, long-haul systems look to transmit enormous data rates over very large distances with as few repeaters as possible. This requires densely packed channels, high input powers and low chromatic dispersion. While FWM is the primary limiting factor, XPM is still considered. The effects of XPM, albeit not as severe, are still substantial. Also, continuing improvements to dispersion compensation technology could potentially allow for higher dispersion fibers in the future, in which case FWM would be less of a problem. This could leave XPM as the primary limiting factor [11, 24].

Several different configurations are compared, both analytically and using simulations. For simplicity only three- and six-channel scenarios are considered. The per channel data
rate for the six-channel scenarios is 10 Gb/s and the per channel data rate for the three-channel scenarios is 20 Gb/s. Hence all systems considered here transmit a total data rate of 60 Gb/s. 10 Gb/s per channel is an especially popular per channel data rate for WDM systems [3, 8].

Whenever a six-channel configuration is compared to a three-channel configuration, the three-channel configuration uses a channel spacing double that of the six-channel configuration. Also, the per channel average power of the three-channel configuration is doubled. Hence the two systems operate with the same bandwidth efficiency and total power.

These restrictions are made so as to ensure fair comparisons between the two. Some of these restrictions are debatable however. In particular, the restriction that the total power be the same for each configuration is not necessarily proper. Unlike many other communications systems, increasing the transmitter power does not in general improve system performance for nonlinear fiber systems. That is, the relation between input power and received eye-size is not a non-decreasing function; the relation is more complicated. It is arguable that in comparing two configurations, each should be assumed to operate with optimal input power. This approach was not used here, due largely to the difficulty of finding the optimal input power for every configuration.

4.4 Theoretical Analysis

The analytic tools available for studying four-wave mixing and cross-phase modulation are limited. Typically theoretical analysis of four-wave mixing is restricted to the continuous wave (CW) assumption [10, 12, 13, 14, 22, 29, 31, 32]. A continuous wave is simply a constant sinusoid, with no amplitude or phase modulation. This is of course useless for transmitting information, but does allow for some insight into four-wave mixing.

Cross-phase modulation can also be analyzed using the CW assumption. However for continuous wave signals XPM simply results in a constant phase shift, which results in no
distortion in IM-DD systems. Because of this numerical techniques are frequently used to explore XPM limited systems [23, 25, 46, 61].

The literature often uses the terminology probe and pump when analyzing WDM systems. The probe refers to the channel of interest and the pump refers to the channel that will induce distortion into the probe channel [8].

### 4.4.1 Four-Wave Mixing

The expression for the power of a four-wave mixing product in an SMF under the CW assumption was first given in [26]. This expression has been widely used and further refined to its present form [10, 14, 22, 29, 32]

\[
P_{klm}(L) = \eta D^2 \frac{\gamma^2}{9} P_k P_l P_m e^{-\alpha L} \left( \frac{(1 - e^{-\alpha L})^2}{\alpha^2} \right) \tag{4.7}
\]

where \(\eta\) is the FWM efficiency, \(D\) is the degeneracy factor, \(\gamma\) is the nonlinear coefficient, \(\alpha\) is the fiber attenuation, \(L\) is the fiber length and \(P_k, P_l, P_m\) are the powers of the three incident channels located at frequencies \(f_k, f_l, f_m\), respectively. The degeneracy factor, \(D\), equals three for a degenerate mixing product and six for a nondegenerate mixing product, see Section 2.2. The mixing product \(P_{klm}\) is located at frequency \(f_k + f_l - f_m\).

The FWM efficiency, \(\eta\), can be expressed as [29]

\[
\eta = \frac{\alpha^2}{\alpha^2 + \Delta m^2} \left( 1 + \frac{4e^{-\alpha L} \sin^2(\frac{\Delta m}{2})}{(1 - e^{-\alpha L})^2} \right), \tag{4.8}
\]

where \(\Delta m\) is referred to as the phase-matching factor. This is typically expressed as [29]

\[
\Delta m = \frac{2\pi \lambda_m^2}{c} \Delta f_{km} \Delta f_{lm} \left( D_C + \frac{\lambda_m^2}{2c} (\Delta f_{km} + \Delta f_{lm}) \frac{dD_C(\lambda_m)}{d\lambda} \right), \tag{4.9}
\]

where \(\Delta f_{km} = |f_k - f_m|\) is the channel spacing in Hz, \(D_C\) is the chromatic dispersion coefficient and \(\lambda_m\) is the wavelength corresponding to frequency \(f_m\). Using the expression for \(D_C\) given in (1.11) and expressing frequency in radians, (4.9) becomes

\[
\Delta m = \beta_2 \Delta \omega_{km} \Delta \omega_{lm} \left( 1 - \frac{\lambda_m}{2\pi c} (\Delta \omega_{km} \Delta \omega_{lm}) \right). \tag{4.10}
\]
For systems with channel spacing less than 100 GHz, \( \frac{\Delta m}{2\pi c} (\Delta \omega_{km} \Delta \omega_{lm}) \ll 1 \). Hence \( \Delta m \) can be simplified to

\[
\Delta m \approx \beta_2 \Delta \omega_{km} \Delta \omega_{lm}.
\] (4.11)

Using the continuous wave assumption, consider a six-channel scenario and a three-channel scenario as shown in Figure 4.1. The six-channel scenario has a channel spacing of \( \Delta \omega \) and the power per channel is \( P_{ch} \). The three-channel scenario has a channel spacing of \( 2\Delta \omega \) and the power per channel is \( 2P_{ch} \). In both cases the center channels are more adversely affected by FWM than the outer channels. The channel \( f_0 \) is exposed to the following FWM products in the six-channel scenario:

<table>
<thead>
<tr>
<th>FWM Product</th>
<th>( \Delta \omega_{km} )</th>
<th>( \Delta \omega_{lm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_1 + f_1 - f_2 )</td>
<td>( \Delta \omega )</td>
<td>( \Delta \omega )</td>
</tr>
<tr>
<td>( f_2 + f_2 - f_1 )</td>
<td>( \Delta \omega )</td>
<td>( \Delta \omega )</td>
</tr>
<tr>
<td>( f_1 + f_1 - f_0 )</td>
<td>( \Delta \omega )</td>
<td>( \Delta \omega )</td>
</tr>
<tr>
<td>( f_3 + f_3 - f_2 )</td>
<td>( \Delta \omega )</td>
<td>( \Delta \omega )</td>
</tr>
<tr>
<td>( f_3 + f_3 - f_1 )</td>
<td>( \Delta \omega )</td>
<td>( \Delta \omega )</td>
</tr>
</tbody>
</table>

Figure 4.1: Continuous Wave Configurations
Channel $f_0$ in the three channel-scenario is subject to the following mixing products:

<table>
<thead>
<tr>
<th>FWM Product</th>
<th>$\Delta \omega_{km}$</th>
<th>$\Delta \omega_{lm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{-1} + f_1 - f_0$</td>
<td>$2\Delta \omega$</td>
<td>$2\Delta \omega$</td>
</tr>
<tr>
<td>$f_1 + f_{-1} - f_0$</td>
<td>$2\Delta \omega$</td>
<td>$2\Delta \omega$</td>
</tr>
</tbody>
</table>

Using (4.7) through (4.11) and the tables above, the total FWM power interfering with channel $f_0$ versus channel spacing and $\beta_2$ is plotted in Figure 4.2. In Figure 4.2.a the x-axis corresponds to channel spacing for the six-channel scenario. The corresponding channel spacing for the three-channel scenario is twice as large.

As can be seen from Figure 4.2, moving from the six-channel scenario to the three-
channel scenario is only beneficial within a limited window. For typical low-dispersion fibers the channel spacing for the six-channel configuration must be between 15 and 50 GHz in order for the three-channel configuration to offer substantial improvement. And for typical DWDM systems the GVD parameter, $\beta_2$, must be between 0.75 and $4 \, \text{ps}^2 / \text{km}$. Within these boundaries switching from the six-channel configuration to the three-channel configuration clearly reduces FWM.

### 4.4.2 Cross-Phase Modulation

While the scenarios of interest here are affected more by FWM than XPM it is still useful to consider the effects of XPM. As mentioned above, CW analysis of XPM is generally not useful. Because of this numerical techniques are frequently used to model the effect of XPM.

Consider a two-channel scenario, with a probe wave, $P_k$, and a pump wave, $P_m$. The probe is continuous wave whereas the pump has an information sequence modulated onto it. An approach similar to the split-step Fourier method can be used to determine the net impact of XPM on the probe due to the pump. The phase shift due to XPM incurred in a short segment of fiber can be modeled independently of other effects. The dispersive nature of the fiber converts this phase distortion to amplitude distortion as the wave propagates through the rest of the fiber. The contribution due to each segment can then be integrated throughout the entire length of the fiber [23].

As with the SSF method, the Fourier transform facilitates the calculation. The frequency domain representation of the change induced in the probe by XPM can be expressed as [23]

$$
\Delta \hat{A}_{km}(\omega, L) = 2P_k(L)\gamma_k e^{\frac{i\omega}{\gamma_k}} \hat{P}_m(\omega, 0) \cdot \left\{ \frac{e^{\frac{1}{2}\beta_2\omega^2 L} - e^{(-\alpha + i\omega d_{km})L}}{i\alpha + \omega d_{km} - \frac{\beta_2\omega^2}{2}} - e^{\frac{1}{2}\beta_2\omega^2 L} - e^{(\alpha + i\omega d_{km})L}}{i\alpha + \omega d_{km} + \frac{\beta_2\omega^2}{2}} \right\} 
$$

(4.12)

where $P_k(L) = P_{ch}e^{-\alpha L}$ is the power of the probe signal at length $L$ if it had been subject
only to fiber attenuation, ignoring dispersion and nonlinearities. \(v_k\) is the group velocity of the probe, \(\tilde{P}_m(\omega, 0)\) is the Fourier transform of the power of the complex envelope of the pump input field and \(d_{km}\) is the relative walk-off between the probe and the pump, which can be approximated as \(d_{km} \approx D_C \Delta \lambda_{km} = \beta_2 \Delta \omega_{km}\). If the assumption of perfect dispersion compensation is made the above expression can be simplified to

\[
\Delta \tilde{A}_{km}(\omega, L) = 2P_k(L)\gamma_k e^{i\omega \tilde{v}_k L} \tilde{P}_m(\omega, 0) \cdot \left\{ \frac{1 - e^{-(\alpha + i\omega d_{km})L}}{i\alpha + \omega d_{km} - \frac{\beta_2 \omega^2}{2}} - \frac{1 - e^{(\alpha + i\omega d_{km})L}}{i\alpha + \omega d_{km} + \frac{\beta_2 \omega^2}{2}} \right\}.
\]

(4.13)

Figure 4.3: Total worst-case XPM power interfering with channel \(f_0\) versus channel spacing (a) or \(\beta_2\) (b). For the six-channel configuration: 25 GHz channel spacing, 5mW peak power per channel. For the three-channel configuration: 50 GHz channel spacing, 10mW peak power per channel.
To calculate the power distortion due to XPM, the inverse Fourier transform is used to convert the spectrum calculated with these equations back into the time domain. Once back in the time domain, either the variance of this distortion signal can be computed or the minimum and maximum can be found to compute the worst-case XPM distortion. The curves given below focus on the later, although the curves generated from the variance yield the same trends.

Using (4.13), four curves are generated. The four configurations are a binary six-channel system, a duobinary three-channel system, a 4-ary three-channel system and a binary three-channel system. The channel spacing and power per channel of the three-channel configurations are double those of the six-channel configuration. Similar to the FWM analysis, the curves show the total worst-case XPM power distortion acting on the center channel, $f_0$.

As Figure 4.3 shows, the six-channel configuration is superior to all of the three-channel configurations at suppressing XPM. Keep in mind the effect of XPM on the scenarios of interest here is dwarfed by FWM however. The magnitudes shown in Figures 4.2 and 4.3 cannot be directly compared because the FWM analysis assumes CW signals whereas the XPM analysis modulates the pump wave.

The most striking feature of Figure 4.3(a) is perhaps the apparent independence of XPM on channel spacing. Within the domain of dense WDM, increasing channel spacing does little to diminish XPM. However, XPM will diminish for much larger channel spacing. Also of note is that XPM decreases as GVD decreases, unlike FWM. This is because XPM in and of itself induces only a phase shift, dispersion is responsible for converting this phase distortion in amplitude distortion.
4.5 Simulations

Numerous simulations were run to compare multilevel signals with conventional binary signals. The focus was largely on moving from a binary six-channel configuration to a multilevel three-channel configuration. A binary three-channel configuration was also considered as well as a duobinary six-channel configuration. Preliminary results showed the 4-ary six-channel configuration to yield virtually identical performance as the duobinary six-channel configuration and hence was neglected.

For six-channel configurations only the performance of the center two channels was measured. And for the three-channel configurations only the performance of the center channel was measured. These channels suffer the most interference and hence will limit overall system performance.

![Figure 4.4: Receiver eye-size versus electronic filter bandwidth, for the center channel of three channel a binary on-off RZ system. 10mW peak power per channel, 20 Gb/s data rate per channel, 30 GHz channel spacing, $\beta_2 = 1 \text{ ps}^2/\text{km}$ chromatic dispersion, 35 GHz optical filter bandwidth.](image)
As described in the previous chapter, the receiver consists of two filters. The demultiplexer acts as a low-order filter and there is an electronic filter after the photodetector. The bandwidths of these filters are frequently chosen to be about 1.75 times the channel bit rate for the optical filter and 0.75 times the channel bit rate for the electronic filter [39]. However, in this research it was found that often these filter bandwidths did not yield optimal performance. In practice, for a given modulation format the bandwidth of each filter is chosen to optimize performance. To make fair comparisons between different configurations, the configurations should not all be restricted to the same filter bandwidths. Optimal filter bandwidths should be used for each configuration. Due to the nonlinear nature of the system it was not possible to predict optimal filter bandwidths, especially for RZ pulses. An example curve showing eye-size versus the electronic filter bandwidth is shown in Figure 4.4. Therefore a search algorithm was used for each simulation point to find near-optimal filter bandwidths.

4.5.1 Results

Results are reported both in terms of minimum eye-size and eye-closure penalty. In all six-channel configurations the per channel peak power is 5mW, and 10mW for the three-channel configurations. All other parameters outlined in Chapter 3 are constant for all simulations. Some example eye diagrams are shown in Figure 4.5. 1536 total information bits were simulated for each configuration. Holding the channel spacing fixed and varying $\beta_2$, the results in Figures 4.6 and 4.7 were obtained.

Figure 4.6 gives the minimum eye-size versus $\beta_2$ for five configurations. For binary pulses the minimum eye-size is calculated using the standard definition. The minimum eye-size is taken to be the average of the two or three separate eyes for duobinary and 4-ary configurations, respectively. Optimal level spacing for multilevel signals is discussed in Section 4.5.3.
Figure 4.5: Example eye diagrams for the input (a) and output (b). The binary and duobinary signal use NRZ pulses and the 4-ary signal uses RZ pulses. Three-channel configuration, 80 GHz channel spacing, 10mW peak power per channel, $\beta_2 = 1 \text{ ps}^2 / \text{km}$ chromatic dispersion.
Figure 4.6: Minimum eye-size versus $\beta_2$ for NRZ pulses (a) and RZ pulses (b). For six-channel configurations: 25 GHz channel Spacing, 5mW peak power per channel. For three-channel configurations: 50 Ghz channel spacing, 10mW peak power per channel.

Of particular interest is the performance of the multilevel three-channel configurations in the 0.75 to 1.5 $\text{ps}^2\text{/km}$ region. Typical dispersion-shifted fibers offer $\beta_2$’s between 1 and 3 $\text{ps}^2\text{/km}$. Hence this is the class of DSF’s most effected by FWM. Both duobinary and 4-ary three-channel configurations demonstrate better performance than binary six-channel. However this improvement is dwarfed by standard binary three-channel. The duobinary six-channel configuration shows consistently inferior performance. A similar picture is seen looking at the eye-closure penalty. Figure 4.7 again shows the binary three-channel configuration to be
Figure 4.7: Eye-closure penalty (dB) versus $\beta_2$ for NRZ pulses (a) and RZ pulses (b). For six-channel configurations: 25 GHz channel Spacing, 5mW peak power per channel. For three-channel configurations: 50 Ghz channel spacing, 10mW peak power per channel.

the most robust. Although for RZ pulse shaping the 4-ary signal shows a smaller penalty for higher $\beta_2$. The multilevel three-channel configurations demonstrated improvement over the binary six-channel configuration for smaller channel spacing, where FWM is most severe.

Holding $\beta_2$ fixed and varying the channel spacing, the results in Figures 4.8 and 4.9 were obtained.
4.5.2 Observations

As can clearly be seen from the figures in the previous section, the standard binary three-channel configuration offers superior performance throughout. This is not surprising given that the multilevel signals must divide essentially the same dynamic range as the binary signal into two or three separate eyes. That is, if the input eye-size for the binary configuration is 10mW then the average eye-size for the duobinary configuration is approximately 5mW and for the 4-ary configuration 3.33mW.
This begs the question, should the configurations be restricted to the same average power, or should each configuration use an optimal power level? Unfortunately there is no simple method for calculating the optimal input power for a given configuration, a search would be required. This would increase simulation time considerably.

Another possible concern for the binary three-channel configuration is that the bandwidth of each individual channel is twice that of the duobinary and 4-ary three-channel configurations. In [39] it was shown that halving the number of channels and doubling the
per channel data rate yielded no improvement in performance. This was because, while the larger data rate system did more effectively suppress nonlinearities, it was subject to greater distortion due to dispersion. [39] was operating in a high-dispersion domain however and was also modeling imperfections in the dispersion compensator and fiber amplifier. Because the simulation used here assumed perfect dispersion compensation, the binary three-channel configuration suffered no performance penalty due to its large bandwidth.

Nonetheless, more than anything else, these simulations demonstrate the utility of moving from a binary six-channel configuration to a binary three-channel configuration when operating with dense channel spacing and low-dispersion. Overall the multilevel configurations did not present any meaningful performance improvement.

### 4.5.3 Optimal Level Spacing

Consider a 4-ary ASK signal with equal-probable field amplitudes given by \(\{0, \varepsilon_1, \varepsilon_2, 1\}\). If the only noise and interference the signal is subject to is stationary, that is, independent of the signal, then \(\varepsilon_1 = \sqrt{\frac{1}{3}}\) and \(\varepsilon_2 = \sqrt{\frac{2}{3}}\) are optimal. The square root is needed because the photodetector outputs a signal proportional to the amplitude squared of the optical signal. Optical systems are subject to both stationary and non-stationary noise sources. If an optical system is limited by noise that is linearly dependent on the optical amplitude, such as shot noise, then \(\varepsilon_1 = \frac{1}{3}\) and \(\varepsilon_2 = \frac{2}{3}\) [55]. The system modeled in this research is noiseless however. The dominant interference, four-wave mixing, is clearly signal dependent. The optimal level spacing was found experimentally to be \(\varepsilon_1 = \left(\frac{1}{3}\right)^\frac{2}{3}\) and \(\varepsilon_2 = \left(\frac{2}{3}\right)^\frac{2}{3}\). The levels were also scaled so that the signal would have the same average power as a binary signal with levels \(\{0, 1\}\). The scaled levels for the 4-ary signal were \(\{0, 0.4708, 0.7917, 1.0733\}\). Similarly for the duobinary signal, the levels used were \(\{0, 0.6436, 1.0824\}\).
4.5.4 NRZ vs. RZ Pulse Shaping

There is no general rule for determining whether NRZ or RZ pulse shaping is preferable for fiber optic systems. From the plots in Section 4.5.1 it is easy to determine whether NRZ or RZ pulse shapes are best for a particular scenario. However, the average power between the two curves is is not the same. The plots all use the same peak power per channel, hence the average power of the RZ signals is half that of the NRZ signals. Note however, that doubling the power of the RZ signals would not necessarily increase the eye-size.

Comparing NRZ and RZ pulses in fiber optic communications is not as simple as many other communication systems. A great deal of research has already been dedicated to the subject and there is still no generally accepted interpretation of the two formats [25, 47, 48, 49, 50, 51, 52, 53, 54].

RZ pulse shaping offers two benefits that are especially useful in WDM systems. The first is that soliton pulses can be constructed from RZ pulses such that the effects of self-phase modulation and chromatic dispersion precisely cancel each other [8, 50]. Another is that for systems with sufficiently low dispersion it is possible, using RZ pulses, to offset each channel so as to minimize the number of adjacent channels that simultaneously have large signal energy [40, 41, 42], this was described in Section 2.3.3.

Even without these techniques RZ is often claimed as superior to NRZ in systems limited by nonlinearities [25, 47, 49, 50, 53, 62]. The philosophy behind most of these claims is that because RZ has a shorter duty cycle there is less time during every bit interval during which FWM and XPM can distort the signal. These works do not consider the dense WDM scenario however, in which the larger bandwidth required for RZ signaling could be a problem.

There is also a substantial body of work contending that NRZ pulses more effectively suppresses FWM and XPM [48, 51, 54]. [52] demonstrates that RZ pulse shaping is more effective for systems limited by nonlinearities, but NRZ pulse shaping is more effective for
systems limited by dispersion. Again, these works were not focusing on DWDM system.

Overall the results found in this thesis might imply that NRZ is more effective at suppressing nonlinearities than RZ in dense WDM systems. However the simulations run here use the same peak power, not average power, so it is unclear whether a direct comparison can be made. Using (4.12) it is possible to show that RZ pulses with the same peak power are more susceptible than NRZ pulses to XPM. It is worth noting that even though the RZ simulations used half the average power as the NRZ simulations, there are still instances where the RZ simulation performed better than the NRZ simulation, such as the 4-ary signal in the low-dispersion region.

4.6 Chapter Conclusion

This chapter presents multilevel signaling in the context of nonlinear fiber optics. It is shown that multilevel signals have potential for alleviating nonlinear effects under some circumstances but that overall standard binary signaling offers superior performance. The next chapter will present another signaling technique with the hope of mitigating nonlinearities, unequal data-rates.
Chapter 5
Unequal Data Rates

5.1 Unequal Channel Spacing

The philosophy used in Chapter 4 is to ameliorate performance by reducing the power of FWM products interfering with information channels. In the mid-1990’s a different approach was proposed, to rearrange the channels so as to reduce the number of FWM products interfering with information channels [12, 30, 31]. This technique garnered much attention throughout the rest of the decade [6, 19, 32, 33, 34, 48]. While this technique has proven useful for suppressing FWM in experimental systems, it has not yet been implemented in any commercial systems.

As described in Section 2.2, four-wave mixing depletes signal energy from three incident channels located at frequencies $\omega_k$, $\omega_l$ and $\omega_m$ to generate a new channel located at

$$\omega_{klm} = \omega_k + \omega_l - \omega_m, \tag{5.1}$$

where $m \neq k, l$ (although $k$ can equal $l$). If the channels of a WDM system are equally spaced many of these mixing products will interfere with the incident channels. The idea proposed in [30] was to assign unequal channel spacing to the $N$ channels in a WDM system such that,

$$\forall \, k, l, m \in \{1 \ldots N\} \ (m \neq k, l), \quad \omega_{klm} \not\in \{\omega_1, \omega_2 \ldots \omega_N\}. \tag{5.2}$$
This technique assumes that the bandwidth of each channel is relatively small compared to the channel spacing. In other words, this technique only applies to WDM systems, not DWDM systems. A simple three-channel case is shown in Figure 2.2, if the channels had been equally spaced some of the FWM products would have interfered with the incident channels.

To satisfy (5.2) the problem is first reformulated. By dividing the available optical bandwidth into bins of equal size, $\Delta \omega_{\text{bin}}$, such that each bin is at least as large as the minimum bandwidth required for each channel, it is possible to reduce the problem to the so called integer linear programming (ILP) problem. It has been shown that the bandwidth of FWM products is only slightly larger than that of the original channels [30, 31], hence each FWM product will also fit into one bin. The total number of bins is much larger than the number of channels, $N$.

The $k^{\text{th}}$ channel is centered around the optical frequency $\omega_k = \omega_0 + n_k \Delta \omega_{\text{bin}}$, where $\omega_0$ is an arbitrary reference frequency and $n_k$ is an integer that will be referred as the bin number. (5.1) can now be rewritten as [30, 31],

$$n_{klm} = n_k + n_l - n_m.$$  \hspace{1cm} (5.3)

The problem now is to assign $N$ channels to the available bins such that $n_{klm}$ does not hit any of the bins in which information channels are present. That is [30, 31],

$$\forall k, l, m \in \{1 \ldots N\} \ (m \neq k, l), \quad n_{klm} \not\in \{n_1, n_2 \ldots n_N\}. \hspace{1cm} (5.4)$$

Let $n_a, a = \{1 \ldots N\}$ represent a bin containing an arbitrary channel. Then for $a, k, l, m \in \{1 \ldots N\}$ and $a, m \neq \{k, l\}$, the above condition can be written as [30, 34],

$$n_a \neq n_{klm} = n_k + n_l - n_m,$$  \hspace{1cm} (5.5)

and thus,

$$n_a - n_k \neq n_l - n_m \quad \text{Q.E.D.}$$  \hspace{1cm} (5.6)
That is, a necessary and sufficient condition for ensuring that no FWM products intersect existing channels is that the channel separation between every pair of channels be unique.

For relatively few channels, less than 16, this problem is readily solved using a computer search. However, this problem is NP-complete, and hence as the number of channels increases finding optimum channel spacing becomes a problem. The ILP problem is an extension of the problem of finding optimum Golomb rulers. A Golomb ruler has \( N \) marks distributed such that the distance between any two marks is unique [31]. In [34] three algorithms, originally intended for finding optical orthogonal codes (OOC), were described for efficiently finding channel allocations when the number of channels is prime.

Once found, this optimal channel spacing presents two concerns however. There is no mechanism to move from an \( N \) channel system to an \( N + 1 \) channel system, the channel allocation for each scenario is completely different. Of more concern is the bandwidth expansion factor (BEF). Consider an \( N \) channel system with uniformly spaced channels. Let the channel spacing be \( \Delta \omega \), hence the total bandwidth required is approximately \( (N - 1)\Delta \omega \). The total bandwidth required by an \( N \) channel system with unequally spaced channels and a \emph{minimum} channel separation of \( \Delta \omega \) is approximately \( \text{BEF} \cdot (N - 1)\Delta \omega \). The precise BEF depends on the bin size, but typical values for 6, 8 and 10 channel systems are 1.5, 1.9 and 2.5, respectively [30, 31, 33]. Perhaps a more useful way of interpreting this situation is that if the two scenarios are required to have the same total bandwidth, the minimum channel spacing in the unequally spaced configuration is \( \frac{\Delta \omega}{\text{BEF}} \).

The ultimate effect of the difficulties outlined above is that the method of unequally spaced channels cannot be expanded to WDM systems with a large number of channels. To partially overcome these difficulties periodic unequally spaced channels have been proposed [19, 32, 33]. This method refers to defining a set of \( N_p \) unequally spaced channels according to the rule above, and then repeating this pattern in the frequency domain as many times as needed until all channels are accounted for. This method essentially allows
for a compromise between FWM and BEF.

5.2 Unequal Data Rates

The development presented in Section 5.1 assumes that the channel spacing is relatively large compared to the bandwidth of each channel. In dense WDM systems this is not the case, that is, there is not enough ‘unused’ space between channels to fill with FWM products.

In designing an unequally spaced WDM system each channel is essentially treated as a CW signal, that is a spectral line. The key idea is then to arrange these spectral lines so as to minimize the number of FWM products that intersect existing channels. In DWDM systems each channel cannot be modeled as a CW, however the power spectrum of each channel contains a set of spectral lines. This presents the question, can the spectral lines within each channel be arranged so as to minimize the number of FWM products that intersect other spectral lines? That is, if several strong FWM products hit a given spectral line, the FWM subsequently generated by this spectral line will be very strong. By arranging the spectral lines such that FWM products do not hit other spectral lines, very strong FWM products could be avoided. This question is explored by assigning each channel a different data rate so as to minimize the number of FWM products that intersect existing spectral lines.

The spectral line component of the power spectral density (PSD) of a digital $M$-ary signal with independent and equiprobable symbols is given by [59],

$$G_{\text{line}}(\omega) = \frac{1}{M^2 T_s^2} \sum_{n=-\infty}^{\infty} \left| \sum_{k=0}^{M-1} \tilde{s}_k \left( \frac{2\pi n}{T_s} \right) \right|^2 \delta \left( \omega - \frac{2\pi n}{T_s} \right),$$

(5.7)

where $T_s$ is the symbol period and $\tilde{s}_k$ is the Fourier transform of the $k^{th}$ pulse shape, $s_k$. $\delta(\omega)$ is the well known delta function, defined as $\delta(0) = \infty$, $\delta(\omega \neq 0) = 0$ and $\int_{-\infty}^{\infty} \delta(\omega) d\omega = 1$. 

A necessary and sufficient condition for all spectral lines vanishing is [59],

$$\sum_{k=0}^{M-1} s_k(t) = 0, \text{ for all } t.$$  \hfill (5.8)

Clearly binary on-off keying does not satisfy this condition. For binary on-off keying (5.7) simplifies to,

$$G_{\text{line}}(\omega) = \frac{1}{4T_b^2} \sum_{n=-\infty}^{\infty} \left| \tilde{s}_1 \left( \frac{2\pi n}{T_b} \right) \right|^2 \delta \left( \omega - \frac{2\pi n}{T_b} \right).$$  \hfill (5.9)

where $T_b$ is the bit period. The relative amplitudes of these spectral lines are of no particular interest here. The key insight is simply that the spectral lines are uniformly spaced with separation $\frac{2\pi}{T_b}$.

Because the spectral lines within any one channel are uniformly spaced, some of the FWM products generated by these lines will hit existing spectral lines. However, by configuring each channel to operate with a different data rate the FWM products generated by spectral lines from different channels can be arranged so as to minimize the number that hit existing spectral lines.

The phenomenon of FWM does not only affect spectral lines of course. A continuum of FWM products distort the entire bandwidth of each channel. However, because FWM is dependent on the power of the three incident waves, the FWM products that result from spectral lines are more troublesome than the FWM products generated by the remainder of the channel.

5.3 Simulations

Numerous simulations were run for various configurations. All configurations shared some traits in common. All were six channel, with 5mW peak power per channel. The total data rate was 60 Gb/s. 1536 bits were simulated for each configuration. Many unequal data rate combinations were explored, all of which gave very similar results hence only one combination is reported here.
In order that the results were not biased by any edge effects, only the eyes of the center four channels were measured. To make fair comparisons this required that the total data rate of the center four channels be 40 Gb/s.

![Graph showing minimum eye-size versus channel spacing for NRZ pulses (a.) and RZ pulses (b.). EDR: equal data rates, UDR: unequal data rates, ES: equal channel spacing, US: unequal channel spacing. All configurations use 5mW per channel peak power. The x-axis gives the channel spacing for the ES configurations.](image)

Figure 5.1: Minimum eye-size versus channel spacing for NRZ pulses (a.) and RZ pulses (b.). EDR: equal data rates, UDR: unequal data rates, ES: equal channel spacing, US: unequal channel spacing. All configurations use 5mW per channel peak power. The x-axis gives the channel spacing for the ES configurations.

Three configurations were simulated using both NRZ and RZ pulses. One configuration with equal data rates, [10, 10, 10, 10, 10, 10] Gb/s, and two with unequal data rates, [12.4, 5.8, 13.9, 9.1, 11.2, 7.6] Gb/s. One of the unequal data rate configurations used equal channel spacing. The other set channel spacing to be proportional to the data rates. That is,
if the channel spacing for the equally spaced configuration was \([50, 50, 50, 50, 50]\) GHz, then the channel spacing for the proportionally spaced configuration was \([45, 48.75, 57.5, 50.75, 47]\) GHz. There is nothing unique about the particular set of unequal data rates used here. Combinations with both more subtle and more extreme differences were simulated, all of which yielded similar results. The most subtle combination employed data rates between 9 and 11 Gb/s and the most extreme combination employed data rates between 2.5 and 17.5 Gb/s.

![Figure 5.2: Eye-closure penalty versus channel spacing for NRZ pulses (a.) and RZ pulses (b.). EDR: equal data rates, UDR: unequal data rates, ES: equal channel spacing, US: unequal channel spacing. All configurations use 5mW per channel peak power. The x-axis gives the channel spacing for the ES configurations.](image-url)
As in the previous chapter, optimal detector filter bandwidths were used for every simulation point. The filter bandwidths for all configurations were proportional to the data rate for each channel. Both average minimum eye-size and average eye-closure penalty are reported.

As can be seen in Figures 5.1 and 5.2, neither of the unequal data rate configurations yield any performance improvement. On the other hand, at least performance does not suffer much by using unequal data rates. This could be reassuring to network designs, where often a flexible network is required to handle channels operating at different data rates. For larger channel spacing, the use of unequal data rates with RZ pulses yields somewhat worse performance compared to equal data rates.

One factor that was not considered when proposing the idea of unequal data rates was the integrating effect of the filters at the receiver. The FWM products due to spectral lines, while substantial, are themselves still spectral lines. Hence their impact on the overall signal energy is not particularly strong. In more conventional communication environments the use of narrow-band filters to remove these isolated frequency distortions is sometimes used. If this technique could be used for WDM systems then moving the FWM products away from the original spectral lines would make them easier to remove. However such narrow-band filters do not exist in the optical domain.

### 5.4 Chapter Conclusion

This chapter explores the use of unequal data rates to partially alleviate FWM in dense WDM systems. The technique of using unequal channel spacing is well established for WDM systems but does not translate into the dense WDM domain. It was found that using unequal data rates does not improve performance significantly. The next chapter presents a third, and final, idea for combating four-wave mixing.
Chapter 6
Multichannel Detection

6.1 Motivation

To date four-wave mixing and cross-phase modulation have essentially been viewed as
signal dependent noise sources [3, 8, 9, 12, 15, 19, 20, 21, 31, 33, 39, 48, 49]. That is, while
FWM and XPM are entirely deterministic phenomena, the interference induced by FWM
and XPM is viewed as containing no information. Hence the only mechanism available to
improve performance in FWM limited systems is to suppress the effect of FWM [8, 10, 14,
31, 33, 35, 36, 37, 39, 45].

A detection theory view of this problem might conclude differently however. Perhaps
the effect of FWM can be alleviated by intentionally incorporating this interference into the
decision rule. To a limited extent this is already done. In binary on-off keying systems, the
decision rule is a simple power threshold, as outlined in Section 3.1.5. Because the effect
of FWM is power dependent the optimal detection threshold is not in the middle of the eye,
but rather shifted down.

Four-wave mixing manifests itself as a performance limiting effect for WDM systems,
where FWM is often viewed as a form of interchannel crosstalk. This begs the question,
can a multichannel detection rule be derived to account for four-wave mixing? To the best
knowledge of this author, this question has not been addressed. The question is somewhat
academic however. Section 1.1 explains that WDM is often the preferred format for high-
capacity optical communications because the electronic systems that need to interface with the fiber only need to operate at the data rate of individual channels, not the total data rate of the system. A multichannel detector would remove this benefit.

6.2 Paired Channels

Nonetheless, the question of multichannel detection is intriguing. Consider a simple two channel scenario. As shown in Figure 2.2(a), when the two channels are simultaneously active, two FWM products are generated on either side of the pair of channels. While the two information channels only directly generate two FWM products, these FWM products then mix with each other and the information channels. This process results in a decaying ensemble of FWM products. Example input and output baseband normalized power spectra for two closely spaced channels are shown in Figure 6.1.

![Input and output baseband normalized power spectrum for a pair of channels.](image)

Figure 6.1: Input and output baseband normalized power spectrum for a pair of channels. 25 GHz channel spacing, 10 Gb/s per channel, 10mW peak power per channel, NRZ pulse shaping, $\beta_2 = 1 \text{ ps}^2 / \text{km}$, 50 km fiber length.
Of interest here are the two largest FWM products, centered at $\pm 37.5$ GHz in the figure. These two FWM products are only generated when both channels are transmitting. For binary on-off keying this equates to the FWM products only being present when both channels are simultaneously transmitting 1 bits. If either channel is transmitting a 0 bit the FWM products are not present.

Figure 6.2: Received eye diagram paths for right FWM eye. Each of the four subplots represents one of the four possible bit combinations of the information channels. Two channels transmitted, 25 GHz channel spacing, 10 Gb/s per channel, 10mW peak power per channel, NRZ pulse shaping, $\beta_2 = 1 \frac{ps^2}{km}$, 50 km fiber length.

If the two channels are sufficiently close together the walk-off effect between the two channels is negligible and the FWM products are relatively large. Hence the frequency bands to the right and left of the pair of channels can be thought of as generating new signals due to FWM. These new signals can be used to generate new eye diagrams, with a positive path representing the instances when both channels are transmitting a 1 bit, and a
zero path otherwise. Figure 6.2 demonstrates this.

Figure 6.2 contains four plots, each representing the paths taken by the right FWM product for the four possible bit combinations of the information channels. From this figure it is clear that the right and left FWM products contain useful information about the transmitted signals. A detection algorithm, rather than detecting the two information signals independently, could detect both signals together and combine measurements taken from the FWM signals to aid in the decision. The situation is more limited than it seems however. The eyes generated by the FWM signals are much smaller and more unstable than the eyes generated by the information signals. This is shown in Figure 6.3.

![Eye diagrams](image)

Figure 6.3: Received eye diagrams for information signals and FWM signals. Two channels transmitted, 25 GHz channel spacing, 10 Gb/s per channel, 10mW peak power per channel, NRZ pulse shaping, $\beta_2 = \frac{1}{50 \text{ km}}$, 50 km fiber length.

The first noteworthy attribute of the FWM signals is that they are smaller by an order of magnitude compared to the information signals. Secondly, the normalized variance of the
paths corresponding to 1 bits of the FWM eyes is larger than that of the information eyes. As described in Chapter 3, the system being modeled here is noiseless. Most noise sources that corrupt WDM systems would effect all four signals equally. As this noise is introduced into the system, the FWM eyes would be rendered useless before the information eyes are noticeably affected. Hence the FWM eyes would be of little use in aiding detection. This would not be so for systems limited by ASE beat noise however, which is proportional to the signal.

### 6.3 Multiple Pairs of Channels

The situation is complicated further by the addition of more channels. Consider a six channel scenario. Channels are arranged in pairs, each pair using a separation of 25 GHz. The pairs are separated by 200 GHz. Input and output baseband normalized power spectra are shown in Figure 6.4.

Comparing Figures 6.1 and 6.4 it is easy to see the overlapping FWM products in the output of the six channel scenario. In addition to this, new FWM products are generated by channels from different pairs. This results in a collapse of the FWM eye diagrams, as seen in Figure 6.5. In particular, the variance of the paths corresponding to both information channels transmitting 1 bits has increased substantially. Simultaneously, the paths corresponding to only the right information channel transmitting 1 bits has risen.

This scenario, with 200 GHz separation between pairs, represents a performance threshold scenario. As the frequency separation between pairs increases the FWM eye diagrams improve, approximating those of Figure 6.2 for a 400 GHz separation between pairs. As the separation between pairs is decreased below 200 GHz the FWM eyes rapidly deteriorate.

Clearly these frequency separations are well outside the dense WDM domain. Comparing this paired-channel scenario to the equivalent equally-spaced scenario shows the immediate superiority of equally-spaced channels, hence a detailed comparison is not provided.
Figure 6.4: Input and output baseband normalized power spectrum for three pairs of channels. 25 GHz channel spacing within each pair, 200 GHz pair spacing, 10 Gb/s per channel, 10mW peak power per channel, NRZ pulse shaping, $\beta_2 = 1 \text{ ps}^2/\text{km}$, 50 km fiber length.

here. The equally-spaced equivalent of the scenario outlined above, in Figure 6.4, would have a channel spacing of 100 GHz. With this relatively large channel spacing the effect of FWM and XPM is modest, resulting in good performance. The minimum eye-size of the information channels for the equally-spaced configuration is approximately 50% larger than the paired-channels configuration.

6.4 Chapter Conclusion

While the system under consideration in this thesis does not show any practical benefit from using a multichannel detector, this approach could prove quite useful for future system design. As mentioned above, to date the philosophy typically used to improve performance in FWM limited systems has been to suppress FWM, common methods for doing this are summarized in Section 2.3. This is true both for device technology research and commu-
communication theory research. In this chapter a novel approach is proposed, that is, to include the effect of FWM in the detection rule. WDM systems are a logical choice for this approach. The preliminary results found here indicate that extensive further research will be necessary to derive conditions under which this approach yields a practical improvement in performance.
Chapter 7

Conclusion

7.1 Summary of Results

This thesis presents the fundamental concepts involved in nonlinear WDM fiber optic communication systems. Throughout this thesis a communication theory approach is used, in contrast to a device technology of theoretical physics approach. Many of the fundamental questions concerning communication system design are still unanswered in the nonlinear fiber domain. Questions as simple as NRZ versus RZ pulse shaping or four 10 Gb/s channels versus one 40 Gb/s channel have no generally accepted answers in nonlinear WDM systems. Due to the nonlinear relationship between system parameters, results found for one particular configuration cannot be easily generalized to other configurations.

The philosophy of this thesis is to explore a collection of classical modulation formats in the domain of nonlinear fiber optics. While the properties of these modulation formats have been well understood for some time in more conventional linear environments, their applicability to nonlinear WDM systems is relatively unknown. Of particular interest in this thesis is the dense WDM scenario, which is proving to be the format of choice for high-capacity long-haul links as well as very popular for network applications.

Three modulation formats are explored, multilevel signals, unequal data rates and multichannel detection. The simulation results do not show an immediate performance improvement for any of these techniques, however each demonstrates its own subtle benefits
The multilevel signals demonstrate perhaps the most promising results here. Both duobinary and 4-ary signals are considered, with comparable results found for each. Depending on what is defined as the equivalent binary configuration to compare the multilevel signals to, multilevel signaling yields either superior or inferior performance. Comparing a three-channel multilevel configuration to a six-channel binary configuration demonstrates a performance improvement when chromatic dispersion and channel spacing are both small. Duobinary signals in particular demonstrate improved performance. On the other hand, comparing a three-channel multilevel configuration to a three-channel binary configuration demonstrates worse performance. The argument for comparing the three-channel multilevel configuration to the six-channel binary configuration is that the per channel bandwidth is the same for each. Under some conditions, not explored in this thesis, the increased channel bandwidth of the three-channel binary configuration could weaken performance of that configuration.

The use of unequal data rates is explored as a natural extension of unequal channel spacing. Unequal channel spacing has proven useful in some circumstances for suppressing four-wave mixing in WDM systems. However, this technique only works when the channel separation is relatively large. Unequal channel spacing cannot be applied to DWDM systems. Unequal data rates are explored to see if any of the improvements found by unequal channel spacing can be translated to DWDM. The results found here show that unequal data rates do not improve performance. At the same time, unequal data rates to not typically weaken performance either, which could be of some consolation to network designs.

Multichannel detection arguably holds the most long-term potential, but demonstrates no immediate improvement here. In theory four-wave mixing is an entirely deterministic effect. Therefore it is possible to define detection rules that take into account the effect of FWM. Practically speaking, elaborate detection rules are simply too computational inten-
sive to be performed at optical speeds, especially if numerous channels are being detected simultaneously. Here, the simplified scenario of just two channels is considered. It is shown that FWM does operate in such a way that could be theoretically incorporated into a decision rule, although the magnitude of the measurement is too small to be immediately useful. Expanding this technique to multiple pairs of channels is not possible within the dense WDM domain.

Given the limited analytical tools available for studying nonlinear WDM systems, exploratory research is just that, exploratory. There are few techniques available to predict the performance of a novel modulation format. This research did not necessarily look to improve system performance, but rather explore the usefulness of some classical modulation formats in the nonlinear WDM domain. In that sense this research is a success. This thesis elucidates, at least partially, third-order nonlinear phenomena in fiber optics and presents some techniques in the hope of mitigating these phenomena. The modulation formats proposed here are found to be somewhat useful for alleviating four-wave mixing, but often by introducing some other performance limiting feature.

7.2 Recommendations

As with any research project, more work remains to be done. While the three modulation formats explored here did not yield a performance improvement for the system of interest, there are perhaps other WDM systems where these techniques will improve performance. Unfortunately, due to the nonlinear nature of fiber optic systems, it is not possible to extrapolate how these modulation formats will perform in other WDM environments.

One parameter in particular warrants extended exploration. This thesis sets constant the total average power between all configurations being compared. While this is often assumed in communication theory research, it is arguably not a proper constraint here. Unlike conventional communication systems, the relationship between transmitted power
and system performance is not monotonic. Typical communications systems are frequently limited by the amount of power that can be transmitted, due either to regulations or device constraints. This is not so in fiber optics, typical laser diodes are capable of producing more than enough optical power. As transmitted power increases the effect of fiber nonlinearities, such as XPM and FWM, quickly becomes catastrophic. Any one configuration will have an optimal transmitting power, although finding this optimal power requires an exhaustive search in general. Perhaps the modulation formats explored in this thesis should be reexamined using an optimal transmitting power for each configuration.

The multilevel signaling configurations could yield a performance improvement. This would most likely involve scenarios where the bandwidth of individual channels is an issue. Multilevel signals have already been shown to improve performance in dispersion limited systems, where the bandwidth compression of multilevel signaling results in less dispersion. The systems of interest here are limited by nonlinearities, hence this bandwidth compression offers no immediate benefit. Even in these systems specific devices, such as fiber amplifiers and dispersion compensation filters, introduce undesirable effects that could be minimized by compressing the bandwidths of each channel.

The use of unequal data rates demonstrates limited potential and further research in the area is not recommended.

Multichannel detection demonstrates long-term potential, but would require extensive further research to yield a measurable performance improvement. In WDM systems perhaps unequal channel spacing coupled with multichannel detection could be used to measure numerous FWM products and detect several channels simultaneously.

In addition to this, many other modulation formats have yet to be explored in nonlinear optics. A brief survey of current literature quickly reveals that only a few modulation formats have been thoroughly explored. This is due largely to device limitations, only certain types of pulses can be generated and detected. At the same time, devices capable of
more exotic modulation format will not be pursued until some benefit can be demonstrated for other formats.
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