Distributed Power Allocation for Multiuser MISO Indoor Visible Light Communications

Jie Lian and Maëté Brandt-Pearce
Charles L. Brown Department of Electrical and Computer Engineering
University of Virginia, Charlottesville, VA 22904
Email: jl5qn@virginia.edu, mb-p@virginia.edu

Abstract—In multiuser MISO (multiple-input single output) visible light communication (VLC) systems, it is essential to find effective means for allocating the transmitted power from the various LEDs fairly among users that are randomly dispersed in large indoor environments. Distributed algorithms are a natural choice because, unlike centralized power allocation algorithms, distributed schemes do not require extensive computation complexity and computation time. In this paper we propose and compare two distributed algorithms: a partially jointly optimized power allocation and a weighted distributed power allocation. The system uses code division multiple access (CDMA) plus optical on-off keying (OOK) with a minimum mean squared error (MMSE) receiver to diminish the effect of multiple-access interference in the indoor VLC system. We compare the performance of the proposed distributed algorithms with an optimal centralized approach. From the numerical results, both distributed algorithms proposed in this paper have less than a 2 dB power penalty compared with the centralized approach, and the computational complexity can be reduced by 75% for a small indoor environment and by 93% for a large indoor environment.

I. INTRODUCTION

With the rapid development of light emitting diodes (LED) technology, visible light communication (VLC) has emerged as a possible technology for next generation communications due to its many advantages compared to radio frequency (RF) communications. VLC systems are environmentally friendly, generate no electromagnetic interference, provide large unregulated spectral resources, and potentially offer higher security. Moreover, VLC systems can be applied in environments that disallow RF, such as airplanes and hospitals [1], [2].

In indoor VLC systems, one significant research challenge that has received some attention in recent years is how to support many users with high data rates while limiting the multiple access interference (MAI). Three directions have emerged to address this problem. One is to use a multiple input multiple output (MIMO) technique with a precoding algorithm to eliminate the MAI [3], [4], and [5]. The second direction is to use color-shift-keying modulation over red-green-blue (RGB) LEDs and CDMA to support multiple users [6]. The third trend is to use a resource allocation scheme to minimize the MAI. For the third trend, the transmitted power allocation schemes for orthogonal frequency division multiple access (OFDMA) and discrete multi-tone (DMT) modulation to eliminate the MAI were proposed in [7] and [8], respectively. Since the peak to average power ratio is relative high for the OFDMA technique, the nonlinear response of the LEDs needs to be considered in the design, adding complexity. In addition, for OFDMA, the phase synchronization needs to be taken into account in the system.

Recently we proposed a joint optimization algorithm we named PAJO (power allocation by joint optimization) to find the power allocation that maximizes the minimum signal to interference plus noise ratio (SINR) among all users in a multiple input single output (MISO) architecture where multiple lamps can support each user [9]. To avoid the nonlinear effect and the phase synchronization needed by OFDM, we use CDMA with optical OOK as our multiple access technique in PAJO, and again in this paper. From the results in [9], the PAJO algorithm has bit-error-rate (BER) performance superior to other schemes. However, as it is a centralized power allocation algorithm, it has high computational complexity and is thus not scalable to large indoor environments. For indoor spaces with many LEDs and users, a distributed scheme with higher computational efficiency is needed.

Two distributed power allocation algorithms with low computational complexity are proposed in this paper. They are called the partial distributed power allocation joint optimization (PD-PAJO) and the weighted distributed power allocation joint optimization (WD-PAJO) algorithms. In the centralized PAJO approach, all the LED lamps need to communicate with each other to perform the power allocation optimization for all the users in the indoor environment jointly [9]. However, for PD-PAJO, only some of the lamps need to exchange information with each other to compute the power allocation. For WD-PAJO, all the lamps work independently. In this paper, an optical CDMA technique is used to make the user signals separable for PD-PAJO and WD-PAJO. A minimum square error (MMSE) filter is used to reject residual multiple access interference at the receiver. We examine the effect of the LED lamp access area on performance. According to our simulation results, PD-PAJO and WD-PAJO have similar BER performance when the per-lamp access area is small. The two proposed distributed algorithms have less than 2 dB transmitted power penalty with the same BER performance compared with the more complex PAJO algorithm.

The remainder of the paper is organized as follows. The system model is described in Section II. In Section III, we derive the PD-PAJO and WD-PAJO algorithms. Numerical results are discussed in Section IV. Finally, the paper is...
concluded in Section V.

II. SYSTEM DESCRIPTION

Typical indoor VLC systems use white LED lamps as transmitters. Unlike RF communication systems, the most viable modulation/demodulation technique for VLC systems is intensity modulation/direct-detection (IM/DD). In this paper, on-off keying (OOK) is used as the underlying modulation scheme for the CDMA signal. For a small indoor space, we use a 25-LED lamp model shown in Fig. 1, where each LED can be controlled separately. For a larger indoor environment, to reduce the computational complexity we use what we call the 1-LED lamp model as the transmitter, shown in Fig. 2. Here the entire lamp (all LEDs) are controlled together as one.

Studies on the impulse response of an indoor optical channel show that the received optical pulse can be classified into a line of sight (LOS) and a diffuse component [10]. For computational complexity reasons, in this paper we assume the channel gain only contains the LOS part. Thus, the general formula for the channel gain between the channel gain between the qth LED and the kth user’s photodetector (PD) can be written as [9]

\[ h_{qk} = \frac{R_c A_r \cos(q_{nk})}{2\pi d_{qk}} (m + 1) \cos^m(q_{rk}, q_k) \]  

where \( A_r \) is the area of the photodetector, \( R_c \) is its responsivity, \( d_{nk} \) is the distance between the LED and the user, \( q_{nk} \) is the unit vector pointing from the LED towards the user, \( q_{rk} \) is the kth receiver’s normal unit vector, and \( l_q \) represents the radiation unit direction vector for the qth LED. In (1), the notation \( \langle x, y \rangle \) represents the angle between vectors \( x \) and \( y \). Fig. 3 illustrates this notation. In addition, \( m \) is the Lambertian mode of the light source, which is related to the LED’s semiangle \( \Phi_{1/2} \) by \( m = \frac{\ln 2}{\ln(\cos(\Phi_{1/2}))} \).

To design distributed power allocation algorithms for VLC, we define for each lamp a circular access area, smaller than its illumination area, as shown in Fig. 4. Only if the user is located in the access area can it be served by this lamp. Our approach to designing distributed resource allocation is to restrict each lamp to transmit data only to a subset of the users in the room, depending on what access areas they are in. Since PAJO is a centralized algorithm solving for the power allocation for all LEDs in the room, it takes all users into account [9].

In the indoor environment, suppose there are \( K \) users in an access area and \( Q \) LEDs serve them, and denote them by \( k = 1, \ldots, K \) and \( q = 1, \ldots, Q \), respectively. Also assume all LEDs are synchronized with each other. The VLC channel between LED \( q \) and user \( k \) is completely characterized by \( h_{qk} \) using (1). Let \( s_k \) be the signal that is intended for user \( k \) at a given symbol time. The qth LED sends a linear combination of the users’ data as [9]

\[ y_k = \mathbf{d}^T \mathbf{B}_k \mathbf{C} \mathbf{s}_k + \mathbf{n}_k^T \mathbf{w}_k, \]  

where \( \mathbf{d} \) is the data vector, \( \mathbf{d} = (d^{(1)}, d^{(2)}, \ldots, d^{(K)})^T \).

We define the intended data for each user to be a binary data stream, i.e., \( d^{(k)} \in \{0, 1\} \). The noise vector is \( \mathbf{n}_k \), which is assumed to be Gaussian and white with variance \( \sigma^2 \), and the MMSE filter for user \( k \) is defined as \( \mathbf{w}_k \). \( \mathbf{C} \) represents the CDMA code matrix, which can be represented as \( \mathbf{C} = (c_1, c_2, \ldots, c_K)^T \), where \( c_k \) is the CDMA code for user \( k \). We chose non-orthogonal CDMA codes to in-
crease the illumination while still maintaining a relatively high power efficiency. To facilitate the formulation, we define \( B_k = \text{diag}(h_{1k}, h_{2k}, \cdots, h_{Q_k}) \), where \( h_k = (h_{1k}, h_{2k}, \cdots, h_{Q_k})^T \). The MMSE receiver \( w_k \) for user \( k \) is obtained as
\[
w_k = (C^T B_k \Sigma_d B_k C + \sigma^2 I)^{-1} C^T B_k q_k, \tag{5}
\]
which is designed independently for each user, where \( \Sigma_d \) is the covariance matrix for the data, \( \Sigma_d = E\{d^T d\} \), \( q_k = E\{d \cdot d(k)\} \), and \( I \) is the identity matrix.

The SINR for user \( k \) can thus be written as [9]
\[
\text{SINR}_k = \frac{w_k^T C^T B_k \hat{E}_k \Sigma_d \hat{E}_k B_k^T C w_k}{w_k^T C^T B_k \hat{A}_k \Sigma_d \hat{A}_k B_k^T C w_k + \sigma^2 w_k^T w_k}, \tag{6}
\]
where \( \hat{E}_k \) is defined as a matrix with a 1 in its \((k,k)\)th element and zeros in all other places, and \( \hat{A}_k = I - \hat{E}_k \).

In this paper we do not consider the illumination requirements, which were considered in [9]. It is straightforward to show that our algorithms in this paper can also work well under specific illumination requirements. Further research on the coexistence of illumination and communication requirements is needed, but is outside the scope of this paper.

III. DISTRIBUTED POWER ALLOCATION ALGORITHMS

In this section, we describe the two proposed distributed power allocation algorithms, PD-PAJO and WD-PAJO. We first derive their operation, and then analyze their computational complexity.

A. Partial Distributed Power Allocation Joint Optimization (PD-PAJO) Algorithm

Since all lamps do not need to allocate power to all users, we consider a partial distributed power allocation joint optimization. Depending on the users’ position, some lamps need to exchange information with each other and work together for power allocation optimization. In this paper, we assume the time differences between the arrival of signal components from different lamps are small enough compared with symbol period to be ignored. The lamps that work together are considered as an optimization thread. Since the threads can be operated in parallel and independently, the computational complexity of PD-PAJO is much lower than that of PAJO. The principles of PD-PAJO are as follow.

For PD-PAJO, the indoor area can be classified into single-covered areas and cross-covered areas shown in Fig. 5 (a). When a user is located in a single-covered area, it can only be served by the lamp whose access area it falls within. But if the user moves to a cross-covered area, all the lamps whose access area include the user transmit to it jointly. In this latter case, all the lamps that serve the same users need to communicate with each other, be in the same thread, and perform the power allocation optimization by maximizing the minimum SINR for all users in this thread jointly. For a thread \( T_i \) this process can be described as
\[
\mathbf{P}_i = \arg \max_{\mathbf{P}} \min_{k \in T_i} \text{SINR}_k \tag{7}
\]
where \( \mathbf{P}_i \) is the power allocation matrix for \( i \)th lamp, \( A_i \) is the access area of lamp \( i \), and \( \gamma_k \) is the number of lamps serving user \( k \).

For example, in Fig. 5 (b), lamp 1 serves users \( a \) and \( b \) but not user \( c \). Since user \( b \) is located in the cross-covered area illuminated by lamps 1 and 2, it is served by these two lamps. Thus lamps 1 and 2 are in thread \( T_1 \). Similarly, lamps 3 and 4 are in thread \( T_2 \). Since the threads \( T_1 \) and \( T_2 \) can operate in parallel, PD-PAJO improves computational efficiency.

For the general case, the PD-PAJO algorithm can be accomplished via the following three-step procedure:

1) The radius of the access area for each lamp is determined.
2) Each user tells the lamps that it is in their access area.
3) Each thread allocates the transmitted power for their lamps in a parallel manner.

Note that since each lamp in PAJO serves all the users in the indoor environment, and the lamps in PD-PAJO only serve the users located in their access area (artificially defined to be smaller than the illumination area), PAJO and PD-PAJO always yield different solutions, even if there is only one thread.

B. Weighted Distributed Power Allocation Joint Optimization (WD-PAJO) Algorithm

For WD-PAJO, the indoor area can be classified into single-covered area, double-covered area, triple-covered area, and so on, as shown in Fig. 6 (a). In this algorithm each LED lamp works independently, and just serves the users that are located in its access area. Thus, there is only one lamp per optimization thread. To eliminate the MAI, each lamp does power allocation optimization through maximizing the minimum weighted SINR for all the users in its own access area. The SINR is weighted by the number of lamps transmitting to the user so that users served by many lamps are not unduly advantaged. For lamp \( i \), the algorithm computes
\[
\mathbf{P}_i = \arg \max_{\mathbf{P}} \frac{\gamma_i}{\kappa} \cdot \min_{k \in A_i} \text{SINR}_k, \tag{8}
\]
where \( \mathbf{P}_i \) is the power allocation matrix for the \( i \)th lamp, \( A_i \) is the access area of lamp \( i \), and \( \gamma_k \) is the number of lamps serving user \( k \).
As the objective functions for the three lamps can be represented the power allocation optimization in parallel. For this example PAJO, all the three lamps must work independently. Since the system, therefore, the CC for PAJO is

where \( N \) is the number of lamps, \( \tau \) is the number of LEDs per lamp, and \( K_{tot} \) represents the number of users in the indoor environment. Obviously, for a large room, many lamps are needed for illumination, and thus the computational complexity is extremely high, especially for large \( \tau \).

PD-PAJO and WD-PAJO are two distributed power allocation schemes with much lower average CC than PAJO. The actual CC depends on the users’ positions in the room. The worst case is that the users are all in the same location and the radius of access area is extremely large. In this case, the order of complexity of PD-PAJO is the same as that of PAJO, which is represented as

and the CC of WD-PAJO is

When the users are uniformly distributed in the indoor environment, and the CC of PD-PAJO and WD-PAJO are functions of the radius of the access area. The CC of PD-PAJO can be approximately represented as

where \( R \) is the radius of the access areas, and \( R_{min} \) is the minimum radius of each access area for all the indoor space to have coverage. The CC of WD-PAJO for this case is given by

Fig. 7 shows an example illustrating the CC of the PAJO, PD-PAJO and WD-PAJO algorithms. For PAJO in Fig. 7 (a), since the four lamps need to communicate with each other and allocate the transmitted power jointly, \( O_{PAJO} = 4 \tau \times 6 = 24 \tau \). Fig. 7 (b) and (c) show how the CC of PD-PAJO and WD-PAJO are \( O_{PD} = O_{WD} = 3 \tau \) in this case. From this simple example, we can see that the proposed PD-PAJO and WD-PAJO have lower computational complexity than PAJO in most circumstances.

### IV. Numerical Results

In this section the performance of the PD-PAJO and WD-PAJO algorithms are shown using simulation. To test the applicability of the algorithms in different environments, we show results for both small and large indoor environments. The parameters used to obtain the numerical results for small and large rooms are shown in Tables I and II, respectively. Fig. 8 shows the geometric position of the lamps and users within the rooms. As for the illumination requirements, the configuration of LED lamps and the position of the light sources can satisfy the 400 lx standard cited in [12], as verified in [9]. A brighter illumination can be achieved by increasing the transmitted power. The access area minimum radius to cover the whole room is \( R_{min} = 1.77 \) m for both scenarios.

For both geometric configurations of the indoor environment considered here, described in Tables I and II, the sum bit rate over all users is taken to be 70 Mb/s per CDMA code reuse.
area, assuming OOK. For larger rooms, the same codes can be used in different parts of the room without interfering with one another and thus each user does not require a unique code. If an M-ary modulation scheme is used instead of OOK, total bit rates in the Gb/s-range can be achieved [13]. For example, if there are 4 code reuse areas in the indoor environment, and 16-ary pulse-amplitude modulation is employed, the total throughput becomes 1.12 Gb/s.

The BER of the proposed PD-PAJO technique for different access area radii in the small indoor environment is shown in Fig. 9 as a function of the peak radiation power to noise ratio (PPNR), which is defined as $P_0/\sigma^2$.\(^1\) We also show the BER performance of the PAJO algorithm for comparison. From the simulation results, when the radius increases from 1.77 m to 2.26 m, the BER of PD-PAJO converges to the PAJO. When $R = 1.77$ m (the minimum radius of the access area), the PD-PAJO has less than a 2 dB power penalty compared with PAJO at a $BER = 10^{-3}$.

Fig. 10 shows the BER of the WD-PAJO technique with different access area radii, also in a small indoor environment, compared with PD-PAJO and PAJO. When the access areas have a small radius, the BER performance of WD-PAJO and PD-PAJO are almost the same, both yielding less than a 2 dB power penalty compared with PAJO. However, as the radius of the access areas increases, the BER curves diverge. The weight $\gamma_k$ for all $k$ in (8) approaches $K_{tot}$, and when all the weight are the same the WD-PAJO algorithm ceases to work well.

\(^1\)Note that in VLC systems we use the transmitted power to receiver noise ratio as an SNR metric, instead of the normal received power to receiver noise ratio.

### Table I

**Parameters Used for Small Indoor Environment**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size of room</td>
<td>5 m × 5 m × 3 m</td>
</tr>
<tr>
<td>Number of lamps</td>
<td>4</td>
</tr>
<tr>
<td>Model of lamp</td>
<td>25-LED model [9]</td>
</tr>
<tr>
<td>Semiangle of LEDs</td>
<td>20°</td>
</tr>
<tr>
<td>Responsivity</td>
<td>0.8 A/W</td>
</tr>
<tr>
<td>Cyclic OOC code index</td>
<td>{1, 2, 4}</td>
</tr>
<tr>
<td>Length of CDMA code</td>
<td>7</td>
</tr>
<tr>
<td>Radius of access area</td>
<td>$R = 1.77 \pm 2.26$ m</td>
</tr>
</tbody>
</table>

### Table II

**Parameters Used for Large Indoor Environment**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size of room</td>
<td>12.5 m × 12.5 m × 3 m</td>
</tr>
<tr>
<td>Number of lamps</td>
<td>25</td>
</tr>
<tr>
<td>Model of lamp</td>
<td>1-LED model [9]</td>
</tr>
<tr>
<td>Semiangle of LEDs</td>
<td>20°</td>
</tr>
<tr>
<td>Responsivity</td>
<td>0.8 A/W</td>
</tr>
<tr>
<td>Cyclic OOC code index</td>
<td>{1, 2, 7}</td>
</tr>
<tr>
<td>Length of CDMA code</td>
<td>25</td>
</tr>
<tr>
<td>Radius of access area</td>
<td>$R = 1.77$ m</td>
</tr>
</tbody>
</table>
Fig. 10. Average BER of 4 users for PAJO and WD-PAJO for different radii of access area in the small indoor environment.

Fig. 11. Histogram of BER performance for 4 randomly distributed users, with peak radiation power to noise ratio of 61 dB.

To analyze the performance of the proposed PD-PAJO and WD-PAJO from a statistical point of view, we simulate 40 trials of 4 users randomly distributed in the small room. From the simulation results shown in Fig. 11, more than 75% of the 40 trials’ BER for PD-PAJO and WD-PAJO are lower than $10^{-4}$. Since some of the lamps in the PD-PAJO algorithm exchange feedback information from the users, while in WD-PAJO there is no information exchange, PD-PAJO is better able to allocate power to users and often results in a lower BER.

The BER performance and the CC of the algorithms depend on the users’ positions. Fig. 12 shows the BER of the worst and best cases for PD-PAJO and WD-PAJO in the small indoor environment. The BER curves for all other cases fall in the region between the best and worst cases. As the radius increases, the BER performance of PD-PAJO improves, while the BER performance of WD-PAJO becomes worse. Considering the computational complexity, the PAJO algorithm has the same CC for all radii. When the radius increases, the CC of both PD-PAJO and WD-PAJO increase. The CC of PAJO acts as an upper bound for the CC of PD-PAJO. The CC of WD-PAJO is upper bounded by $O_{PAJO}/N$. Taking the BER performance and CC into account, we should choose the minimum access area radius. The final design would need to balance these metrics with lighting quality.

The CC and the PPRN required to achieve a BER of $10^{-3}$ for PAJO, PD-PAJO and WD-PAJO are shown in Fig. 13 for the large indoor environment. The users, shown in Fig. 8, are selected in a successive manner. As the number of users increases, the CC of PAJO increases sharply. The CC of the proposed PD-PAJO and WD-PAJO are much lower than the CC of PAJO, and increase slowly. When the BER is $10^{-3}$, the PPRN of PD-PAJO and WD-PAJO are both less than 2 dB higher than the PPRN of PAJO. That is to say, the proposed PD-PAJO and WD-PAJO have to transmit less than 2 dB extra power compared with PAJO to get a BER performance of $10^{-3}$. 
V. Conclusion

In this paper we present two distributed power allocation algorithms we call PD-PAJO and WD-PAJO for a multiuser indoor VLC network. The system uses a CDMA technique with an MMSE receiver for MAI rejection. From the simulation results, PD-PAJO and WD-PAJO algorithms have much lower computational complexity than the optimal centralized PAJO. In addition, PD-PAJO and WD-PAJO have similar BER performance when the access area radius is small. The PD-PAJO outperforms the WD-PAJO when the radius is large, with a corresponding increase in complexity. Compared to PAJO, PD-PAJO and WD-PAJO have less than a 2 dB power penalty to obtain the same BER performance as PAJO.

REFERENCES