Medium Access Control—Physical Cross-Layer Design

Goran Dimić,
Nicholas D. Sidiropoulos,
and Ruifeng Zhang

The layering principle has long been a staple of network design, enabling the modular development of network layers in essential isolation. This piecemeal approach has served us well but has now reached the point of diminishing returns. A consensus towards cross-layer design has been gradually forming as a result of this realization, championed in part by signal processing research. Cross-layer design aims at coupling the functionality of network layers, with the goal of boosting system-wide performance. The trend is more evident at the interface between the physical (PHY) and medium access control (MAC) layers. Signal processing for communications researchers and practitioners are naturally at home with physical layer issues, but less so with queuing, statistical multiplexing, and routing issues that are prevalent at the MAC, data link, and higher network layers. For example, there is a tendency to think that time-division multiple access (TDMA) is somehow optimal for heavy symmetric loads; this is incorrect, because it neglects the randomness of arrivals. This article is meant to unveil multiaccess queuing and introduce useful stability analysis tools to the signal processing readership. A recently introduced class of joint MAC-PHY designs is used to illustrate concepts and tools, and ALOHA is also invoked as a familiar textbook example.

The History of Layering

The layering principle of network design has served for some 30 years as a convenient means of isolating the functions of the different conceptual layers for the purposes of modular design and teaching the principles of complex networking systems. This is so despite the fact that the open systems interconnection (OSI) standard hierarchy never

quite made it into an actual design blueprint. Yet it has recently been recognized that cross-layer design can lift significant performance barriers associated with the modular approach (e.g., [1], [12]). For example, the functionality of the data link layer can be used to effectively improve the physical layer and vice versa. A rudimentary example of this is multipacket combining in the context of automatic repeat request (ARQ) protocols; but more sophisticated uses of the link layer functionality can be envisioned.

Scenario

Consider a system consisting of J transmitting nodes (also referred to as *terminals*) communicating packets to a central access point, e.g., the base station (BS) for the cellular uplink [Figure 1(a)]. For brevity, we focus on a synchronous narrowband system. Time is slotted at the packet level, and all J transmitting nodes are synchronized to slot timing. Nodes do not cooperate. The BS monitors channel access and broadcasts control signals to all nodes. Each node has a buffer in which the incoming packets are stored. Packet arrivals are random and independent across nodes; the vector of average arrival rates of the terminals is denoted by $\boldsymbol{\lambda} := [\lambda_1, \ldots, \lambda_J]^T$, with elements measured in packets/slot. The sum of individual average arrival rates, $\sum_{j=1}^{J} \lambda_j$, is called the *overall traffic load*.

It is well known from queuing theory that, under independent Poisson arrivals, the best utilization of slots and the lowest mean delay per packet is achieved when all the queues are effectively combined in one queue to be served by the appropriate queuing discipline [e.g., first-come, first-serve (FCFS)] [2]. This is known as statistical multiplexing and is readily implementable for downlink transmission, wherein all buffers are physically colocated at the BS. Efficient multiplexing is much more complicated for the uplink, wherein three principal classes of MAC are currently in use: fixed allocation (FA), exemplified by TDMA; random access (RA), with slotted ALOHA being the textbook example; and reservation-based (RB) schemes, e.g., reservation-based TDMA (RB-TDMA). The choice depends largely on traffic characteristics and the sophistication and speed of the available feedback/control mechanism. FA schemes are generally used for "regular" (quasi-deterministic) traffic with limited buffering or strict delay/expiration constraints or where service isolation is critical. RA schemes are generally preferred when arrivals are bursty, the overall traffic load is limited, and low mean delay is paramount. RB schemes bridge the gap between FA and RA; they usually employ RA for the reservation phase.

RA protocols stem from ALOHA [3]. The simplified idea in RA schemes is that each terminal may transmit "at will," whenever it has a packet in the buffer. At light traffic loads with bursty data sources, it is unlikely that two or more will transmit at the same time; hence slots are efficiently used and delay is much smaller than that of
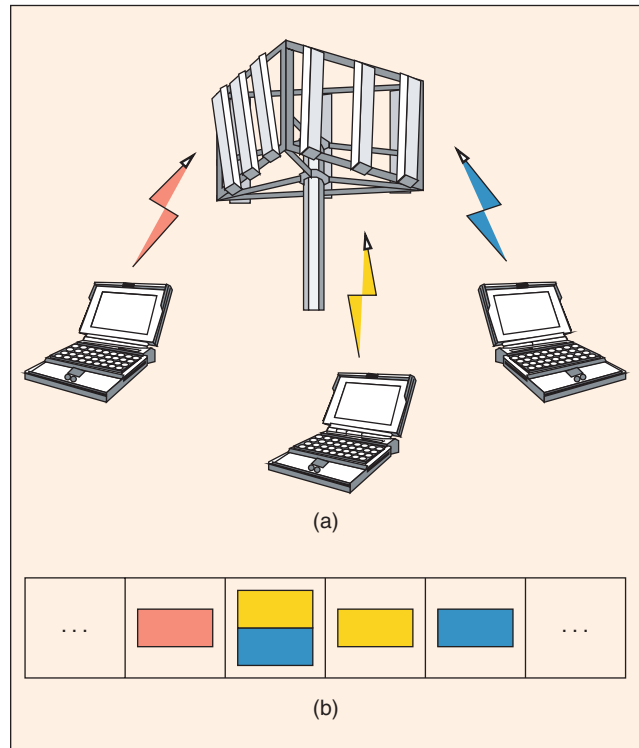
▲ 1. (a) Multiple access in a cellular wireless setup: noncooperating nodes transmit packets to the BS. (b) Collision and its resolution: yellow and blue packets collide; in backoff mode, both are successfully transmitted.

fixed allocation schemes. At higher loads, however, collisions do occur and slots are wasted. Figure 1(b) shows a collision of two packets followed by new transmissions of the same packets. Note that each node retransmits with probability p unaware of other nodes. Hence, instead of the situation depicted in Figure 1(b), secondary collision of the same two packets may occur or it may be that neither of the packets is retransmitted, so that (potentially many) more slots are wasted. This leads to relatively low maximum throughput (average number of successfully transmitted packets per slot) and excessive delay of RA protocols under even moderate traffic load. This collision penalty can be alleviated, to the extent permissible by feedback speed and complexity considerations.

Irrespective of the specifics of a particular solution, there are a few key questions that permeate the subject matter. Given a MAC protocol, what are the operating limits on $\boldsymbol{\lambda}$ to maintain finite buffer sizes and mean delay? What is the mean delay and delay variance for a given $\boldsymbol{\lambda}$? These issues are addressed by queuing theory. We begin with an overview of available RA solutions, then turn to queuing tools and associated stability and delay analysis.

Collision Resolution for RA Protocols

When a collision occurs, the nodes involved in the collision must retransmit their packets in subsequent slots in backoff mode. Retransmission is controlled by the MAC protocol, and it provides means for collision resolution

(CR): the eventually successful transmission of all collided packets. An ideal MAC protocol would resemble statistical multiplexing—there would be no collisions and no wasted slots. It would maintain all buffer sizes finite

Distributed Coin-Toss Resolution of Two-Packet Collision

To gain a first-order intuitive understanding of the tree algorithm, consider a collision of two packets. Each of the two packets is independently retransmitted in the next slot with probability 1/2, while the remaining terminals enter backoff mode. Successful transmission of one packet then happens with probability 1/2; the other packet can then be transmitted in the following slot. In this case, two slots are used for CR. A collision or an idle slot following the initial collision happens with probability 1/2, in which case each of the two packets will be transmitted in the following slot independently with probability 1/2. This procedure can be repeated until the successful retransmission of both packets, at which point CR is complete. With the above strategy, the two-packet collision can be resolved in two slots with probability 1/2, three slots with probability 1/4, and i slots with probability $1/2^{i-1}$. This yields an average of three slots for CR. Including the slot of the original collision, a total of four slots are needed to successfully carry two packets. The resulting efficiency is 50%. Higher-order collisions entail higher number of CR slots/packet, on average, compared to two-packet collisions; hence 50% efficiency is not attainable in general.

(a)

(b)

▲ 2. Binary tree representation of the resolution of a four-packet collision.

provided that the overall traffic load is less than 1 packet/slot. Assuming that all packets are successfully transmitted, we say that such a protocol has maximum stable throughput equal to 1 packet/slot. In this section, we are interested in maximum stable throughput of existing MAC protocols. A more rigorous treatment of the notion of stability will be provided later.

MAC Layer Solutions

The throughput/delay performance of plain slotted ALOHA is often not satisfactory, and more sophisticated access schemes are needed. (See "Determination of Stable Throughput for Slotted ALOHA" and Figure 4 for an illustration of the delay performance of slotted ALOHA.) MAC-layer CR methods focus on coordinating the retransmissions of collided packets such that secondary collisions can be mitigated. Depending on feedback assumptions, several improvements of basic ALOHA relying on MAC-layer functionality have been proposed and are currently in use.

Tree/Stack Algorithms

The first CR algorithm is the tree algorithm [4]–[6]. It assumes ternary feedback (0: empty slot, 1: successful transmission of a packet, e: collision) that is made available to the terminals immediately after the packet transmission is complete. After receiving the e feedback, terminals not involved in the collision switch to backoff mode, whereas those nodes that had transmitted split into two subgroups. Note that, under ternary feedback, a node involved in a collision cannot know the identities or even the number of other nodes involved in the collision.

The tree algorithm can be described using a binary tree as exemplified in Figure 2. The root of the tree is the initial collision group. Each node in the tree represents an empty slot, a successful transmission, or a collision. The collision nodes split into left and right branches (subgroups). Splitting can be either statistical (each node randomly chooses to be in the first or the second group with probability 1/2) or designed a priori, e.g., using the node's binary ID. The left branch corresponds to the first subgroup which transmits after the collision, and the right branch corresponds to the second subgroup that transmits *after the first group completes CR*. In Figure 2, red and blue retransmit in the first subgroup, collide and split again, followed by two successful retransmissions. Yellow and green retransmit and collide, then both back off and again collide before successful retransmissions. It can be seen that the resolution of the original collision group is actually a depth-first search of the binary tree. With 0, 1, e feedback, each terminal can construct the collision resolution tree and thus determine when to transmit its packet. The binary addressing scheme applies only to a finite-user system, while the random addressing one is capable of dealing with the infinite-user case.

For example, the tree depicted in Figure 2 would result if the node IDs were 100 (blue), 110 (red), 001
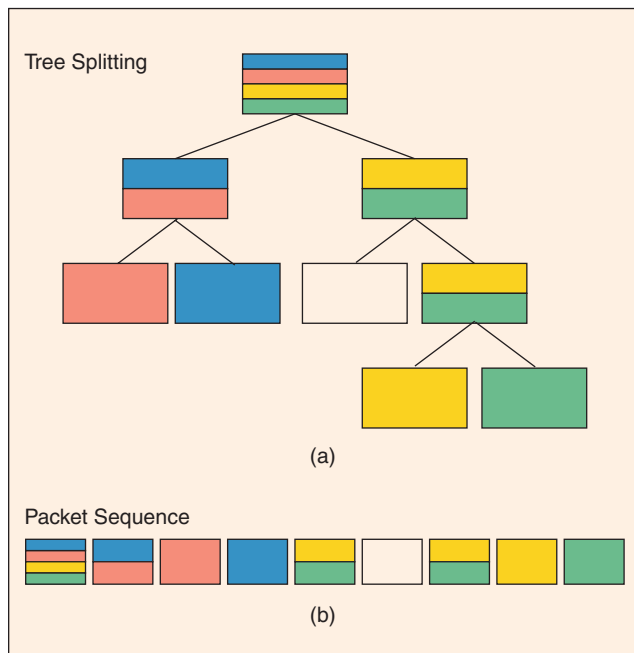
(yellow), and 000 (green). After the initial collision, blue and red go to the left subgroup and both transmit immediately, because their first bit is 1; yellow and green go to the right subgroup and wait for the CR of blue and red to be resolved. Since blue and red are both enabled, they collide again and then split once more. Because the second bit of blue is 0, it goes to the right subgroup and remains silent, while red, whose second bit is 1, transmits successfully. This implies that red was alone in its subgroup and is observed (via the 0/1/e feedback, which is 1 in this case) by blue. Blue now knows it can transmit; it does so, and its transmission is successful. Yellow and green have been observing 0/1/e feedback in the meanwhile. They know that the left subgroup has split once, and two successful transmissions have occurred. Thus nothing remains in the left subgroup, and they can now proceed. Both of them immediately transmit and collide. They look into their second bits, both go to the right (silent) group, and an empty slot occurs. Then both transmit (since both were assigned to the second subgroup) and hence collide again; looking at their third bit, they now go into separate subgroups, and the full collision is resolved after two slots.

The stable throughput of the tree splitting algorithm as described above is 0.43 packets/slot [4], compared to 0.36 packets/slot for slotted ALOHA. However, while slotted ALOHA suffers from bistability, the tree splitting algorithm is stable if the overall traffic load is less than 0.43. Instead of a fixed binary tree, a traffic-dependent dynamic tree is used in the dynamic tree (DT) algorithm to split the collision group into j subgroups where j is chosen to minimize the expected length of the CR period based on the previous CR periods [4]. An interesting feature of the DT algorithm is that it embodies TDMA as a special case when the splitting factor j is fixed and equal to the (finite) number of the nodes. In this way, the DT algorithm offers a graceful transformation from slotted ALOHA at low loads to TDMA at high loads. (Note that for random arrivals TDMA is not optimal at high loads.) On the other hand, the DT algorithm requires online estimation of the average arrival rate for each node (problematic for bursty sources) and adaptation of the retransmission probabilities, which implies a relatively complex control mechanism. Also, DT delay performance is poor at high loads.

Various improvements to the tree algorithm have been proposed. For example, when an idle slot follows a collision, that means that all terminals whose packets collided are assigned to the second subgroup. If all the terminals from the second subgroup transmit after the idle slot, a collision will definitely happen. This can be avoided by letting the second subgroup immediately split into two subgroups and only enable the first of the newly created subgroups to transmit [7]. This improvement can bring the maximum throughput of the tree algorithm up to 0.46 packets/slot. Another improve-

ment is the FCFS algorithm. The idea of the FCFS algorithm is to split the colliding packets according to their arrival time: packets arriving earlier than a predetermined instance go to the first subgroup, and packets arriving later go to the second subgroup. Continuously splitting this way, it leads to successive transmission of packets in the order of their arrival. The FCFS algorithm achieves throughput of 0.4878 packets/slot.

Determination of Stable Throughput for Slotted ALOHA

Consider a symmetric slotted ALOHA system with J terminals. Each terminal is equipped with an infinite buffer and chooses to transmit in a slot with probability p, provided its buffer is nonempty. Packet arrivals are stationary with mean rate λ packets/slot per terminal. Feedback is 0/1/e (idle/success/collision), made available at the end of each slot. Also consider a fictitious system that is identical to the above (including drawing from a common sequence of "coin flips" to determine transmissions), except that terminals may choose to transmit even when their respective buffers are empty, in which case they simply transmit a dummy packet. If both systems are started from the same initial state and fed with the same arrivals, then the queues in the fictitious dominant system can never be shorter than the queues in the original system. This is so because a packet scheduled for service by a queue in the dominant system can never leave the said queue prior to its mirror packet in the corresponding queue of the original system. It follows that if the dominant system is stable, then so must be the original system.

The key here is that the service process for each queue in the dominant system is stationary—something that is not obvious in the original system. Loynes' theorem [19] states that if the arrival process and service process of a queue are stationary, and the average arrival rate λ is less than the average service rate μ, then the queue is stable. This yields that the dominant and hence also the original system is stable provided $\lambda < p(1-p)^{(J-1)}$. The right-hand side (rhs) can be maximized by setting $p = 1/J$. This yields $\lambda < (1/J)(1 - 1/J)^{(J-1)}$ or $\lambda J < (1 - 1/J)^{(J-1)}$. Note that the rhs is a decreasing function of J, starting from 0.5 for $J = 2$ terminals, and quickly (and reassuringly) converging to $1/e$ as $J \to \infty$. For unbuffered infinite-population slotted ALOHA, the textbook derivation of this number is not sharp (e.g., [2]). Drawing heavily from Tsybakov and Mikhailov and Rao and Ephremides, [24], [25], the present approach does not need the Poisson assumption or independence, and it shows that $1/e$ throughput is indeed achievable for any symmetric finite-population slotted ALOHA system with stationary inputs.

Under the aforementioned ternary feedback assumptions, the maximum throughput of RA systems is upper bounded by 0.587 packets/slot [8].

An alternative way of achieving higher throughput is by means of *collision multiplicity feedback*, wherein the number of collided packets is made available to the transmitting terminals at the end of the packet transmission. This can be exploited to optimize the retransmission probability, but delay performance remains poor at higher loads, because collisions are still wasteful.

Carrier Sensing/Collision Detection
Another improvement comes through the use of carrier sensing and collision detection, as in wireline Ethernet local area networks. In carrier sense multiple access with collision detection (CSMA/CD), terminals sense the common bus for a carrier before transmitting, and then listen for collisions during transmission [9], [10]. If a collision is detected, the transmission is immediately aborted. If propagation delay is small relative to packet duration, CSMA/CD alleviates the impact of collisions. This differentiates CSMA/CD from ALOHA-type access, which assumes that feedback is made available *after* packet transmission is complete. A drawback of CSMA/CD is that it is difficult to implement reliably in channels with fading and shadowing (the "hidden terminal" problem) and relatively large propagation delays, which is a typical scenario in cellular wireless systems.

Determination of Stable Throughput for Unslotted ALOHA

The line of argument used in "Determination of Stable Throughput for Slotted ALOHA" can be employed for unslotted ALOHA. Transmissions are now asynchronous, but if a coin flip prohibits a terminal from transmitting, the next coin flip occurs T seconds later, where T is the packet duration (backoff is needed to avoid deadlock). Now the service rate of each queue in the dominant system is $p(1-p)^{2(J-1)}$, because each one of the other queues, backlogged continuously in the dominant system, effectively partitions the time axis in slots of length T, each corresponding to a coin flip. The partitions of different terminals are not synchronized, thus each terminal has two chances to interfere with a given transmission of a queue of interest. Hence $\lambda < p(1-p)^{2(J-1)}$ is needed. Maximizing yields $p = 1/(2J-1)$, and substituting back gives $\lambda < (1/(2J-1))[1-(1/(2J-1))]^{2(J-1)}$ or $\lambda J < (J/(2J-1))[1-(1/(2J-1))]^{2(J-1)}$. For $J = 2$ this yields $(2/3)^3 \approx 0.296$; throughput converges to the familiar $1/(2e)$ for unslotted ALOHA as $J \to \infty$. We see that this simple dominant system approach (coupled with Loynes' powerful theorem) is simpler to digest and accept than the traditional equilibrium arguments employed in many texts.

Reservation Schemes
RB schemes like RB-TDMA bridge the gap between RA and FA. RB-TDMA splits the channel time into a relatively short reservation phase followed by a longer contention-free payload phase. RA schemes are typically used for reservation, but polling is also viable for small terminal populations. To be effective, RB schemes require relatively smooth (as opposed to bursty) traffic. In addition, the use of reservation does not eliminate the problem of multiple access under varying traffic conditions—it shifts it to the reservation channel. The RA protocol used for the reservation phase is an important factor that affects overall system performance.

PHY Layer Solutions
The problem of packet collisions can also be solved at the physical layer using signal processing techniques. The idea is to separate colliding packets at the signaling level, without the involvement of the MAC layer.

Capture
So-called *signal capture* effects come into play in packet transmission over fading channels, whenever one packet is received at a much higher power than others in the colliding group, and thus it can be successfully decoded. The potential of exploiting the capture effect to improve the performance of random access systems has been intensively studied under various fading environments, and modulation and coding schemes. However, capture remains a random effect that cannot always be relied upon.

Multiple Packet Reception (MPR)
Advanced signal processing techniques offer even more effective solutions to CR. Since packet collision can be viewed as a signal mixing problem, it can be tackled in the framework of signal separation. In the communications community, closely related problems have been studied in the context of multiuser detection and antenna array processing.

MPR relies on receive diversity [11], [12]. To resolve a collision, multiple independent collision signal mixtures are needed. This diversity can be made available through spread spectrum. From the communication system point of view, this comes at the price of bandwidth expansion proportional to the spreading gain. Furthermore, higher-order collisions are still possible with any fixed amount of spreading, which limits throughput at high loads. A different twist is to use multiple receive antennas at the access point. This provides a conceptually equivalent multipacket reception capability, which however is practically limited to a small number of antennas ("equivalent chips"). A nice tutorial by Tong et al. on ALOHA networks with MPR capability can be found in [13].

Joint MAC-PHY Signal Processing Solutions: NDMA and BNDMA
A novel approach to the CR problem has been proposed recently in [1]. It is motivated by the diversity

reception concept borrowed from spread-spectrum and receive antenna diversity but explores MAC layer functionality (in particular, the retransmission capability) to build up diversity at the physical layer. The end result can be viewed as a "spreading on demand"-equivalent, wherein demand depends on the collision multiplicity determined by the BS. We collectively refer to this class of protocols as xNDMA; NDMA stands for network diversity multiple access.

Consider again the usual slotted ALOHA model with 0/1/e feedback made available at the end of each channel slot. In the xNDMA context, 0 clears all terminals for transmission, whereas 1 enables those that transmitted in the previous slot and disables all others. Note that each terminal knows whether it has transmitted or not in the previous slot. "e" signals error in packet recovery. When the BS detects a collision it sets feedback to 1, which induces another collision of the same packets in the following slot. After a sufficient number of retransmissions, the BS sets feedback to 0. The slots used for the first transmission and subsequent retransmissions comprise a CR *epoch*.

After a T slots long CR epoch, the PHY layer discrete-time baseband-equivalent data model is

$$X_{T \times N} = A_{T \times K} S_{K \times N} + W_{T \times N}. \qquad (1)$$

N denotes packet length in symbols, and it is fixed. K is the number of collided packets at the beginning of each CR epoch, which is a random variable. S is the collided packet signal matrix, whose kth row is the packet of the kth terminal. A is the mixing matrix, whose entry $[A]_{t,k}$ denotes the complex gain of the symbols of the kth packet during the tth transmission. It can represent either channel fading or a random phase offset due to asynchronism between transmitter and receiver. X is the received data matrix, and W is the white Gaussian noise matrix.

The model in (1) is a classical signal separation problem, where S is to be estimated from X, coupled with a model selection problem, where the number of collided packets, K, has to be determined. Hence, CR proceeds by first detecting K, then determining the appropriate T, and subsequently estimating A. Given the estimated A, S can be estimated using the maximum likelihood estimator or a simpler suboptimal linear estimator [1], $\widehat{S} = \widehat{A}^{\dagger} X$, where $(\cdot)^{\dagger}$ stands for pseudoinverse, provided that A is full column rank. Hence, $T \geq K$ transmissions are needed. Estimation of A can be either training based (NDMA) or blind (BNDMA). The choice between these two approaches determines the appropriate strategy for detecting K. We briefly review these approaches in the sequel.

Training-Based Collision Resolution: NDMA

A terminal identifying (ID) sequence known to the BS can be embedded in the header of each packet. A simple collision detection mechanism is possible if orthog-

onal IDs are used and the BS employs matched filters corresponding to these IDs [1]. Then, the joint terminal detection problem can be decoupled into independent single terminal detection problems, and K set to the total number of detected users. By solving the model selection problem, IDs of terminals involved in collisions are recognized, so that A can be estimated using these IDs.

NDMA is feasible if the channel between every user and the BS in (1) is frequency flat and block fading: constant over each packet slot but different from slot to slot [1]. In slowly fading environments, this can be achieved by multiplexing several NDMA protocols or frequency hopping [14]. In a frequency selective channel, orthogonality can be maintained by using complex exponential ID sequences and a cyclic prefix or guard time; cf. [15].

Note that $T = K$ transmissions are needed to ensure full rank of A in a fading channel, when K packets collide (1). In the finite signal-to-noise ratio (SNR) environment, two kinds of errors may result in packet corruption. The BS may incorrectly determine the collision multiplicity, and hence schedule an insufficient number of retransmissions. This results in a failure to recover some or all of the packets. Even if the collision multiplicity is correctly determined, bit errors may occur due to noise. In [16], ARQ control is suggested to cope with the effects of finite SNR.

Blind Collision Resolution: BNDMA
A blind method for estimating A is motivated by the fact that the length of orthogonal IDs must be equal to or greater than the number of terminals. For large terminal populations this reduces the effective throughput.

For slowly varying channels, it is possible to use a simple retransmission scheme to bypass this limitation and at the same time ensure full column rank of the effective mixing matrix. The trick is to employ exponential phase modulation at the packet level.

The method suggested in [17] produces a mixing matrix A with Vandermonde structure, while preserving the same PHY data model (1). This is achieved using the following retransmission scheme: Before the first retransmission, each user randomly draws a digital carrier for the packet, ω_k (for the kth user); In the $(t-1)$th retransmission (tth transmission), the kth user's carrier is multiplied by $(t-1)$, and the whole packet is multiplied by $e^{j(t-1)\omega_k}$. Hence, the entries of A are $[A]_{t,k} = e^{j(t-1)\omega_k}$. Random selection of digital carriers ensures that the resulting Vandermonde mixing matrix has full rank with probability 1. Frequency selectivity can be accommodated with the inclusion of some slot guard time with no other modifications. Channel equalization can be performed after CR, on a single-user basis. This model structure allows use of an ESPRIT-like signal separation method for blind packet recovery [18], [17]. Collision multiplicity is estimated at the BS using rank detection of the received data autocorrelation matrix [17]. When this matrix is deemed full rank, the BS asks for another retransmission. Otherwise, it clears the channel and proceeds with estimating A and S. Note that the collision multiplicity method needs $T = K + 1$ slots to detect rank deficiency in the noiseless case. This is also the condition for the use of the ESPRIT-like method for packet separation, so that a K-fold collision is resolved in $K + 1$ slots in BNDMA.

Queuing Tools

Define the vector of queue lengths (the number of packets in each buffer) at the beginning of the kth transmission period, denoted by $\mathbf{s}(k) := [s_1(k), \ldots, s_J(k)]^T$. A transmission period is usually a slot, but it can also be defined as a CR epoch, e.g., for xNDMA protocols. The question we want to answer is this: For what values of the vector of arrival rates, $\boldsymbol{\lambda}$, the queue lengths and mean packet delay remain stable in some sense? To make this specific, we need to introduce pertinent models for the packet arrivals and associated notions of stability.

Arrival Models and Stability

We distinguish two classes of arrival models:
▲ Poisson arrivals, which are independent across terminals. This is the stochastic model most commonly employed in this context. It is appropriate for traffic entering a network, aggregated from many low-rate users.
▲ (So-called) *Deterministic fluid* arrivals. This model places deterministic constraints on the arrivals, which can also be viewed as hard constraints on all sample paths of stochastic arrivals. This is called the deterministic fluid model, and is appropriate for leaky-bucket rate-controlled packet streams (e.g., traversing interior network nodes).

Based on the choice between stochastic or deterministic arrivals, two corresponding notions of stability can be naturally formulated. We shall first address the probabilistic interpretation of stability, assuming Poisson arrivals. The definition we adopt originated from the work of Loynes [19]. In the form below, it can be found in [20] and [21].

Definition 1
Queue j of the system is *stable*, if

$$\lim_{k \to \infty} Pr\{s_j(k) < x\} = F(x) \quad \text{and} \quad \lim_{x \to \infty} F(x) = 1. \quad (2)$$

$$\text{If } \lim_{x \to \infty} \lim_{k \to \infty} \inf Pr\{s_j(k) < x\} = 1, \quad (3)$$

the queue is *substable*. A stable queue is also substable. If a queue is not substable it is unstable. The system is stable if all the queues are stable. If at least one queue is unstable, the system is unstable. In (2), inf stands for the greatest lower bound.

The definition of stability in (2) is equivalent to the positive recurrence of the associated embedded Markov chain. In other words, the system is stable if and only if there is a positive probability mass function of $\mathbf{s}(k)$ when k tends to infinity. Substability (3) means that different initial conditions $\mathbf{s}(0)$ may yield different positive probability mass functions of $\mathbf{s}(k)$ when k tends to infinity.

An example of the associated embedded Markov chain is that of xNDMA protocols [1], [17]. Setting the transition times at the beginnings of CR epochs, we have

$$s_j(k+1) = \begin{cases} s_j(k) - 1 + n_j(k), & s_j(k) > 0 \\ n_j(k), & s_j(k) = 0 \end{cases} \quad (4)$$

for $j = 1, 2, \ldots, J$, where $n_j(k)$ is the number of new arrivals into queue j during the kth CR epoch. $n_j(k)$ is a Poisson random variable.

The deterministic interpretation of stability follows from the arrivals model first suggested by Cruz [22], [23], in which packet arrivals are assumed to satisfy certain deterministic constraints along each sample path. In this model, the number of packet arrivals to the jth queue over the time interval (s, t), denoted by $n_j(s, t)$, satisfies

$$n_j(s, t) \le \rho_j(t - s) + \sigma_j, \quad t > s. \quad (5)$$

Note that the slope ρ_j serves as an upper bound on the long-term average arrival rate λ_j. In determining *sufficient* conditions for stability, we may set $\rho_j = \lambda_j$. $0 \le \sigma_j < \infty$ is a measure of burstiness [22]. Intuitively, a data stream is allowed to have a burst of σ_j packets, as long as the long-term average rate remains $\le \rho_j$. To see this, divide by $(t - s)$ and take limit as $(t - s) \to \infty$.

In the deterministic fluid context, stability means that every queue in the system remains deterministically bounded for all time.

Dominant System Approach

The dominant system approach was initially developed to overcome the intractability arising in the analysis of the inseparable multidimensional Markov chain for finite-population buffered slotted ALOHA [24]–[26], [20], [21]. The approach is best illustrated by example. Two such examples are presented in "Determination of Stable Throughput for Slotted ALOHA" and "Determination of Stable Throughput for Unslotted ALOHA" (see also "Necessity of the Derived Conditions"), dealing with a plain-vanilla version of the classical ALOHA family of RA protocols.

For slotted ALOHA, it has been shown in [24] that for $J = 2$, $\sqrt{\lambda_1} + \sqrt{\lambda_2} < 1$ is necessary and sufficient for stability; for $\lambda_1 = \lambda_2$ this reduces to $2\sqrt{\lambda} < 1 \Rightarrow \lambda < 1/4$, for total load < 0.5, as per the simpler dominant system argument. For general J, the issue of stability of asymmetric slotted ALOHA is still an open question. A single necessary and sufficient stability condition is missing for $J > 3$. We note that it is possible to obtain other dominant systems which yield a tighter bound for the stability region; on this matter, see [25], [20], and [21].

The dominant system approach can be used to find a sufficient condition for stability of the xNDMA system. Consider a dominant system in which every one of the J queues always transmits one packet at the beginning of a CR epoch, even if it has none in its queue, in which case it transmits a dummy packet. This increases the service time for all queues without affecting arrivals. Queues in the dominant system can be analyzed separately. The CR epoch length is always the same, so a dominant system queue is equivalent to a slotted M/D/1 queue with service time J slots (NDMA), or $J + 1$ slots (BNDMA). The service process is deterministic in the dominant system, hence trivially stationary (note that this is—again—not obvious in the original system). Since the arrivals are stationary as well, Loynes' theorem applies and yields $\lambda_j < 1/J$ for NDMA and $\lambda_j < 1/(J + 1)$ for BNDMA.

Foster-Lyapunov Approach

The Foster-Lyapunov approach is based on the Lyapunov approach to stability analysis of nonlinear systems. We reproduce the Foster-Lyapunov criterion for ergodicity of a Markov chain (e.g., [27]): Suppose that the chain is irreducible and let S_0 be a finite subset of the state space S. Then the chain is positive recurrent if for some $V : S \to \mathbb{R}$ and some $\epsilon > 0$ we have $\inf_q V(q) > -\infty$ and

$$\sum_{w \in S} p_{qw} V(w) < \infty, \quad \forall q \in S_0,$$
$$\sum_{w \in S} p_{qw} V(w) \leq V(q) - \epsilon, \quad \forall q \notin S_0,$$

where p_{qw} is the probability of transition from state q to state w.

An Application of the Foster-Lyapunov Approach

Let us prove that $\sum_{j=1}^{J} \lambda_j < 1$ is sufficient for stability of the NDMA system. Consider the Lyapunov function $V(\mathbf{s}) = \sum_{j=1}^{J} s_j$, defined on the state space $\mathbf{S} = \mathbb{Z}_+^J$ of the Markov chain, where $\mathbb{Z}_+$ denotes the nonnegative integers. For all $\mathbf{s} \in \mathbf{S}$, we have $V(\mathbf{s}(k)) \geq 0$, which satisfies the first condition. For the second condition, we have

$$E[V(\mathbf{s}(k+1)) \mid \mathbf{s}(k)] = E\left[\sum_{j=1}^{J} (s_j(k) - 1\{s_j(k) > 0\}\right.$$
$$\left. + n_j(k)) \mid \mathbf{s}(k)\right]$$
$$= V(\mathbf{s}(k)) - \sum_{j=1}^{J} 1\{s_j(k) > 0\}$$
$$+ E\left[\sum_{j=1}^{J} n_j(k) \mid \mathbf{s}(k)\right] < \infty,$$

because the third term is always finite due to Poisson arrivals, $n_j(k)$, and bounded epoch length ($1\{\cdot\}$ is the indicator function). Consider the last condition. We have

$$E[V(\mathbf{s}(k+1)) - V(\mathbf{s}(k)) \mid \mathbf{s}(k)] = E\left[\sum_{j=1}^{J} (s_j(k+1) - s_j(k))\right.$$
$$\left. \mid \mathbf{s}(k)\right]$$
$$= \sum_{j=1}^{J} E\left[n_j(k) - 1\{s_j(k) > 0\}\right.$$
$$\left. \mid \mathbf{s}(k)\right]$$
$$= \sum_{j=1}^{J} \lambda_j l(k)$$
$$- \sum_{j=1}^{J} 1\{s_j(k) > 0\}$$

where $l(k)$ is the kth epoch length in slots, and it is equal to the number of transmitted packets $\sum_{i=1}^{J} 1\{s_i(k) > 0\}$. Hence, $E[V(\mathbf{s}(k+1)) - V(\mathbf{s}(k)) \mid \mathbf{s}(k)] = l(k)(\sum_{j=1}^{J} \lambda_j - 1)$. Let $\mathbf{S}_0 = \{\mathbf{0}\}$. Then, $\forall \mathbf{s}(k) \notin \mathbf{S}_0$, we have that $l(k) \geq 1$. Therefore, $\forall \mathbf{s}(k) \notin \mathbf{S}_0$, if $\sum_{j=1}^{J} \lambda_j < 1$ then there exists ϵ, where $0 < \epsilon < 1 - \sum_{j=1}^{J} \lambda_j$, such that

$$E[V(\mathbf{s}(k+1)) - V(\mathbf{s}(k)) \mid \mathbf{s}(k)] < -\epsilon. \qquad (6)$$

It follows that all conditions of the Foster's criterion are satisfied. Therefore, we conclude that $\mathbf{s}(k)$ is ergodic. Hence, $\sum_{j=1}^{J} \lambda_j < 1$ is sufficient for stability.

If a chain is irreducible and aperiodic positive recurrent, then it is ergodic (e.g., [27]), which implies (2). Note that $\sum_{w \in S} p_{qw} V(w) = E[V(s(k+1)) \mid s(k) = q]$, and $\sum_{w \in S} p_{qw} V(w) - V(q) = E[V(s(k+1)) - V(s(k)) \mid s(k) = q]$ also. The right-hand side of the last equation resembles the familiar notion of drift of a one-dimensional discrete Markov chain. Then, roughly speaking, one may think of Foster's criterion as a generalization of drift analysis: if for all states that yield large enough value of the Lyapunov function (states outside a finite subset S_0) drift is negative (cf. the last condition), then the size of the queues decreases on average, so that the chain is stable.

A relaxed condition on the arrival rates that guarantees stability of an NDMA system can be obtained by using the Foster-Lyapunov approach, see "An Application of the Foster-Lyapunov Approach" (cf. [28]). Another good example of an application of the Foster-Lyapunov criterion can be found in [29].

Deterministic Fluid Arrivals Approach

The aim of the deterministic fluid arrivals approach is to prove that, under a suitable condition on the λ_j's, the state (backlog) of every queue in the system will remain bounded. One example of use of this elegant approach can be found in [30]. The method usually entails an induction argument, starting by showing stability of the queue with the lowest arrival rate.

In the case of BNDMA, it is easier to prove that every queue in the system will empty out in finite time ad infinitum, irrespective of initial conditions and the state of other queues in the system. This is again achieved using an induction argument and in turn implies that every queue remains bounded [28]. The result is that the BNDMA system is stable if

$$\sum_{j=1}^{J} \lambda_j + \max_{j=1}^{J}(\lambda_j) < 1. \tag{8}$$

The fluid approach is exemplified in "An Application of the Deterministic Fluid Arrivals Approach," applied to a $J = 2$-user BNDMA system; for general J see [28].

Figure 3 depicts stability bounds for a two-user system for slotted ALOHA, NDMA and BNDMA, obtained using the reviewed approaches. Transmission of dummy packets in the dominant system enforces equal service rates of all queues. This reduces the maximum individual rate making the bound loose at points of highest asymmetry—when one terminal occupies the channel. The deterministic fluid and Foster-Lyapunov approaches, on the other hand, capture the asymmetry of arrival rates and yield tighter bounds. The difference in the stability regions of NDMA and BNDMA stems from the extra retransmission required by BNDMA.

Other Performance Analysis Tools

The considered queuing tools generally yield sufficient conditions for stability. Necessary conditions can sometimes be produced from steady-state analysis, e.g., see

[28] for xNDMA. Steady-state analysis assumes stability and typically boils down to *balance equations* on the average number of packets arriving to and departing from a queue, as exemplified in Little's theorem, e.g., [2].

Steady-state analysis methods are also useful in characterizing the effect of noise on the joint MAC-PHY design. This is an important issue when practical PHY layer components are considered. In the context of xNDMA, the estimation of the number of collided nodes is obtained from noisy data. In addition, bit errors may arise from the collision resolution procedure. These make the balance equation of queue arrivals and departures statistically dependent on the signal-to-noise ratio [16].
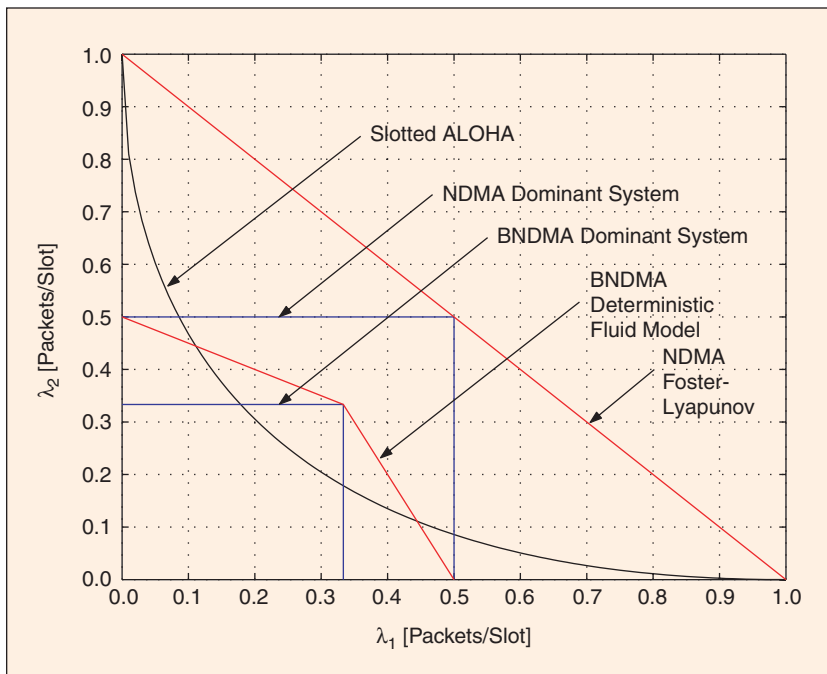
Additional tools are needed to evaluate delay performance of different protocols. These methods are presented in classical books on networking, like [2]. Examples of application of approximate delay analysis tools for Poisson arrivals in the context of xNDMA can be found in [1], [17], and [28]. The basis of their approach is delay analysis of an M/G/1 queue with server vacations. Examples of delay analyses for deterministic fluid arrivals model can be found in the original work of Cruz [22], [23]. Comparison of delay versus throughput performance of TDMA and different RA protocols for a ten-user system with equal arrival rates is presented in Figure 4.
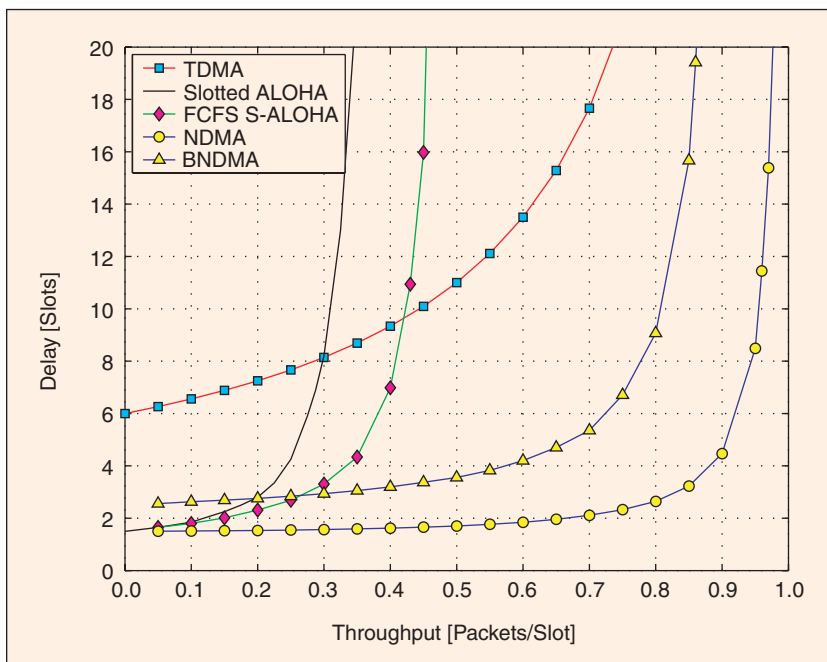
Conclusions

This article has reviewed various aspects of MAC-PHY cross-layer designs, with the goals of introducing pertinent queuing and stability analysis tools to the SP-oriented readership, and illustrating the advantages of such cross-layer designs using the recently introduced class of xNDMA protocols as an example. The results of queuing analysis show that a jointly designed MAC-PHY layer provides increased throughput (close to 1 packet/slot) and low delay characteristics over a wide range of offered loads. The throughput-delay benefits of xNDMA come at the expense of a moderate increase in receiver complexity and power consumption.

Acknowledgments

Goran Dimić received the Diploma in electrical engineering from the University of Belgrade, Belgrade, Serbia and Montenegro, in 1999, and the M.S. degree from the University of Minnesota, Minneapolis, in 2001. He is currently working towards the Ph.D. degree at the University of Minnesota. His research interests are in the area of signal processing for communications and networking.

▲ *3. Two-user stability bounds.*

▲ *4. Delay versus throughput.*

Nicholas D. Sidiropoulos received the Diploma in electrical engineering from the Aristotelian University of Thessaloniki, Greece, and the M.S. and Ph.D. degrees in electrical engineering from the University of Maryland at College Park, in 1988, 1990 and 1992, respectively. He is currently a professor in the Telecommunications Division of the Department of Electronic and Computer Engineering at the Technical University of Crete, Chania–Crete, and adjunct professor at the University of Minnesota. His current research interests are in signal processing for communications and multiway analysis. He is a member of the IEEE Signal Processing Society Technical Committes on Signal Processing for Communications and Sensor Array and Multichannel Processing. He is an associate editor for *IEEE Transactions on Signal Processing* and was an associate editor for *IEEE Signal Processing Letters.* He received the NSF/CAREER award (Signal Processing Systems Program) and a 2001 IEEE Signal Processing Society Best Paper Award. He is a Senior Member of the IEEE.

Ruifeng Zhang received the B.S. degree from Huazhong University of Science and Technology in 1993, the M.E. degree from Beijing Institute of Technology in 1996, and the Ph.D. degree from Stevens Institute of Technology, all in electrical engineering. He is an assistant professor in the Electrical and Computer Engineering Department of Drexel University. His research interests include the areas of statistical signal processing and communications. He chairs the joint SP/BT/CE Chapter, IEEE Philadelphia Section, and is on the organizing committee for ICASSP 2005. He received the 2002 IEEE SP Society Best Paper Award, the Peskin Award from Stevens Institute of Technology in 2000, and AT&T and the ACM Student Research Award in 1999. He is a Member of the IEEE.

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