Frugal Sensing and Estimation over Wireless Networks

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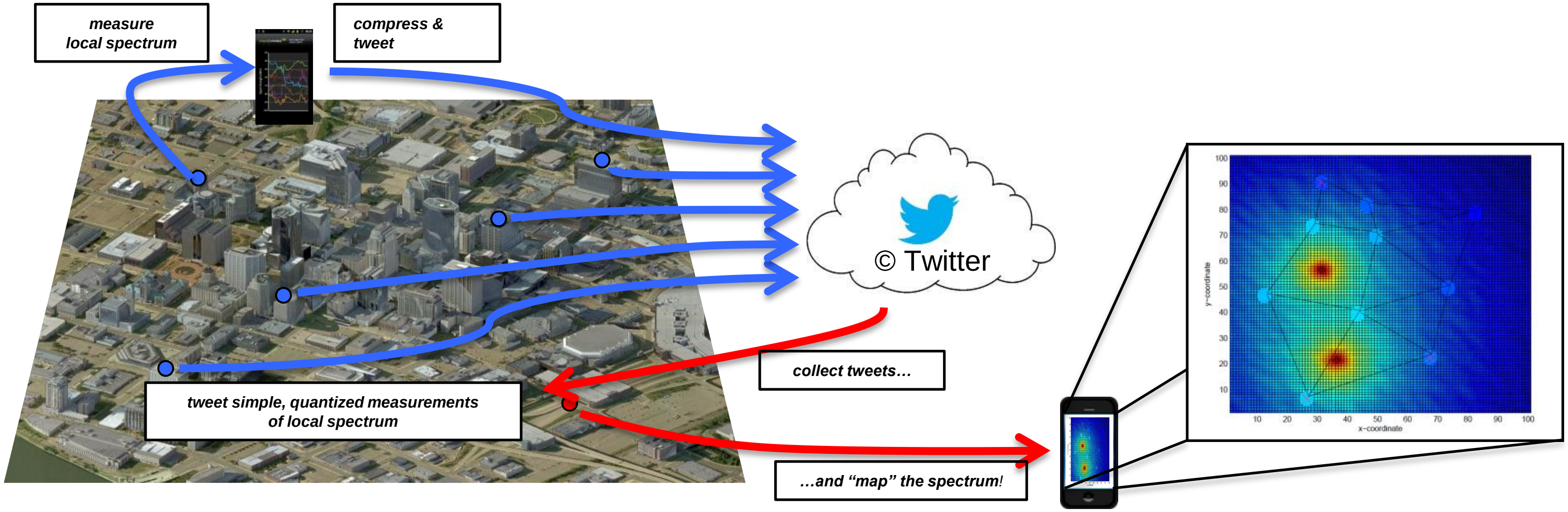
Driven to Discover℠

Frugal Power Spectrum Sensing

- ❑ Motivation:
  - ➢ Crowdsourced spectrum sensing using smartphones



measure local spectrum

compress & tweet

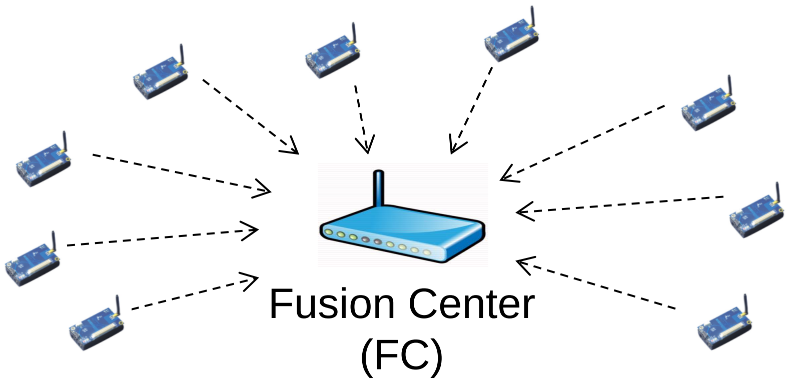
tweet simple, quantized measurements of local spectrum

collect tweets…

…and "map" the spectrum!

- ❑ At the confluence of three areas:
  - ➢ Spectral analysis
  - ➢ Optimization
  - ➢ Distributed detection and estimation

Wireless Sensing Networks



- ❑ Practical limitations
  - ➢ Sensors are battery-operated with limited power, limited transmission bandwidth
  - ➢ Sending analog-amplitude (finely-quantized) vector measurements to the FC is a heavy burden
- ❑ Objective
  - ➢ Develop bandwidth- and energy-efficient strategies
  - ➢ How can the FC detect / estimate / track the signal of interest from (very) few received bits?

Outline

1. Frugal Sensing: Wideband power spectrum sensing from few bits
   - Non-parametric passive sensing
     - Linear programming (LP) formulation
     - Maximum likelihood (ML) formulation
   - Non-parametric active sensing
     - Cutting plane formulation
   - Parametric passive sensing for MA models
     - Non-convex QCQP formulation

2. Frugal Channel Estimation and Tracking for Transmit Beamforming (originally planned; decided to skip)

Frugal Sensing

Wideband Power Spectrum Sensing From Few Bits

Wideband Spectrum Sensing

- Cognitive radio: secondary users scan wide frequency band to identify spectral opportunities

- Wideband sensing
  - High-resolution, high-speed, ADC
  - Hard to implement, expensive, high power consumption

- Multiband sensing
  - Divide into narrowband channels + channel-by-channel sensing
  - Large number of bandpass filters, ignores correlation across bands

- Compressive sensing [Tian-Giannakis’07, Candes’06, Donoho’06]
  - Sub-Nyquist sampling
  - Requires frequency-domain sparsity

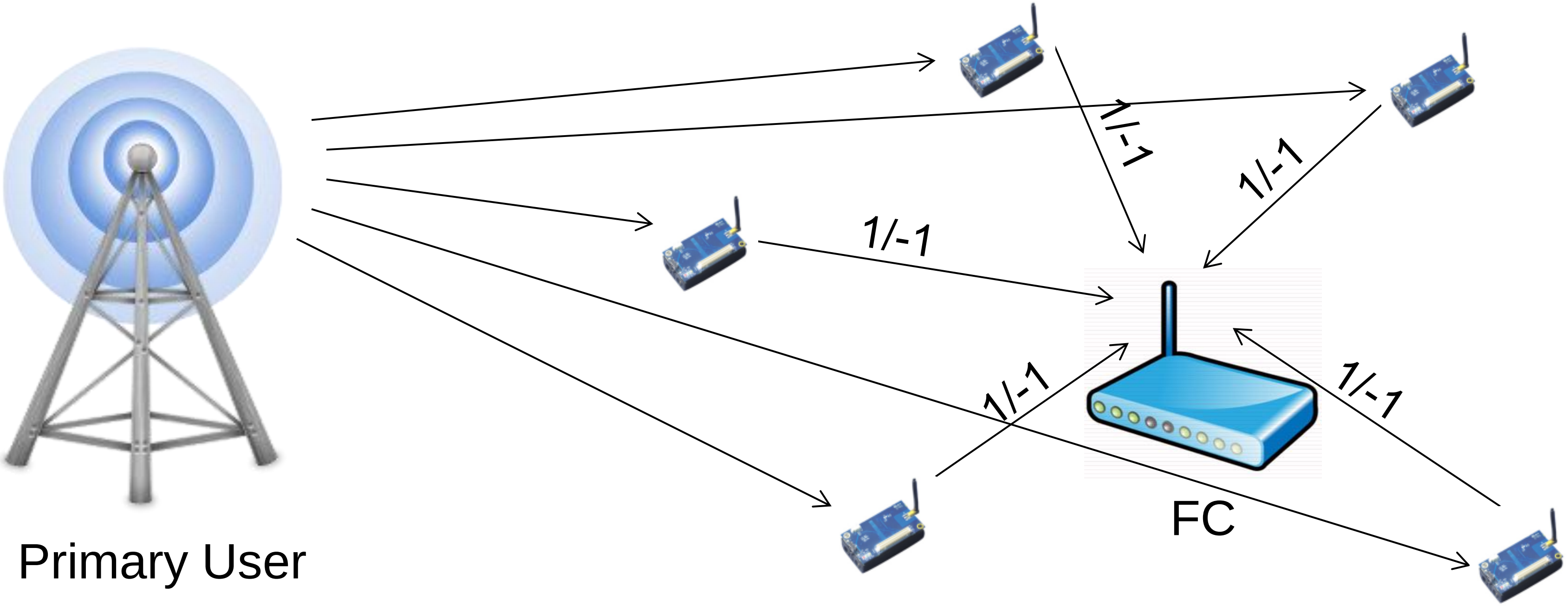
Power Spectrum Sensing

- Only power spectrum (PSD) is needed in many sensing applications (e.g. cognitive radio, radio astronomy)
  - No need to reconstruct the spectrum of the original signal
  - Estimated from Fourier transform of truncated autocorrelation → finite parameterization
  - Sampling rate requirements significantly decreased without requiring frequency-domain sparsity [Ariananda-Leus’11, Lexa-et-al’11]

- Collaborative spectrum sensing
  - Reliable sensing exploiting spatial diversity of sensors
  - Opens the door for crowdsourcing spectrum sensing using today’s smart phones and other wireless devices

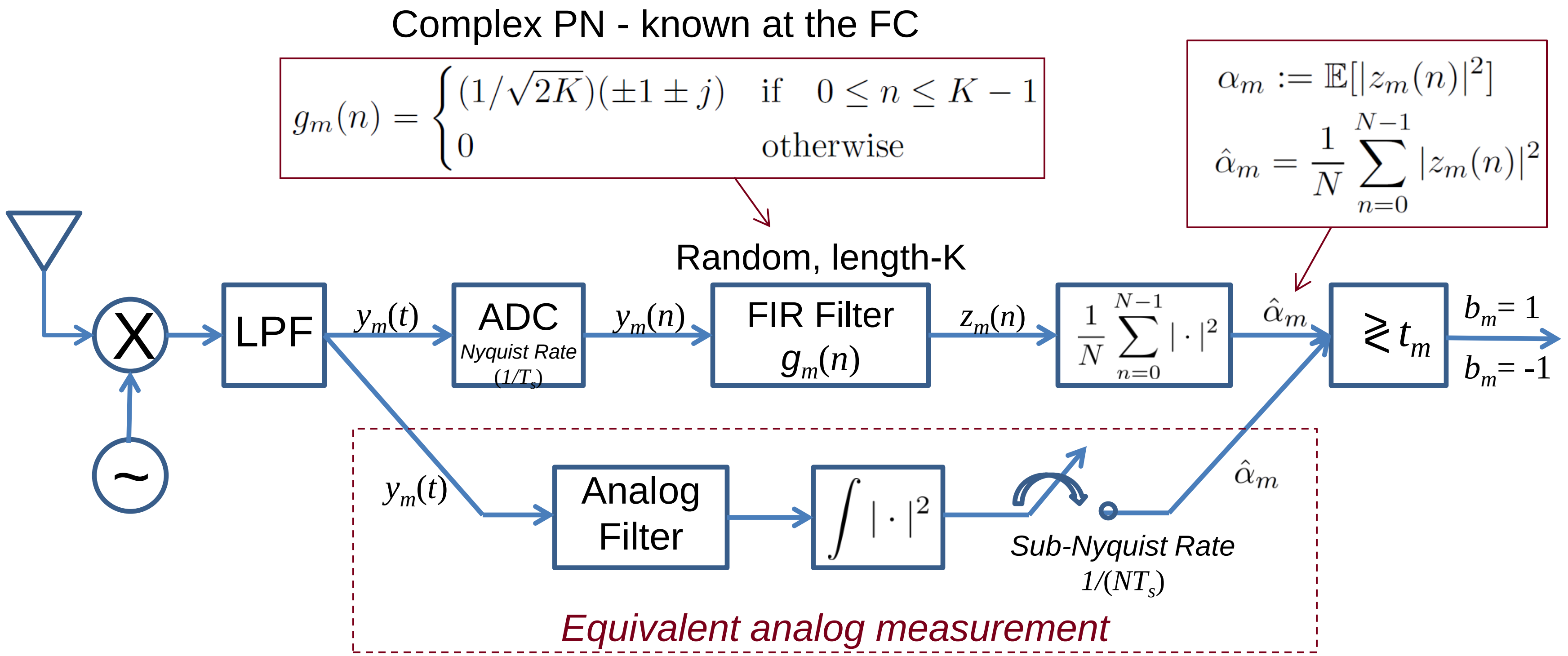
- Challenge: collaborative wideband power spectrum sensing using low-end sensors with limited communication capabilities

Frugal Sensing



Power spectrum estimation from very few bits

Sensor Measurement Chain



Complex PN - known at the FC

$$g_m(n) = \begin{cases} (1/\sqrt{2K})(\pm 1 \pm j) & \text{if} \quad 0 \le n \le K-1 \\ 0 & \text{otherwise} \end{cases}$$

$$\alpha_m := \mathbb{E}[|z_m(n)|^2]$$

$$\hat{\alpha}_m = \frac{1}{N} \sum_{n=0}^{N-1} |z_m(n)|^2$$

Random, length-K

X

~

LPF

$y_m(t)$

ADC

Nyquist Rate

(1/T_s)

$y_m(n)$

FIR Filter

$g_m(n)$

$z_m(n)$

$\frac{1}{N}\sum_{n=0}^{N-1} |\cdot|^2$

$\hat{\alpha}_m$

$\gtrless t_m$

$b_m = 1$

$b_m = -1$

$y_m(t)$

Analog Filter

$\int |\cdot|^2$

$\hat{\alpha}_m$

Sub-Nyquist Rate

$1/(NT_s)$

Equivalent analog measurement

- Random wideband filters
  - Provide independent / complementary views of the underlying PS
  - Better than narrowband filters
    - Narrowband measurements affected by failure/fading
    - No sensor coordination: who covers what, add/remove sensors without reprogramming

Power Measurements

- Received signal at sensor *m*

$$y_m(n) = \sum_{\ell=0}^{L-1} h_m(\ell) x(n-\ell)$$

L-tap fading channel
(frequency-selective)

primary
WSS signal

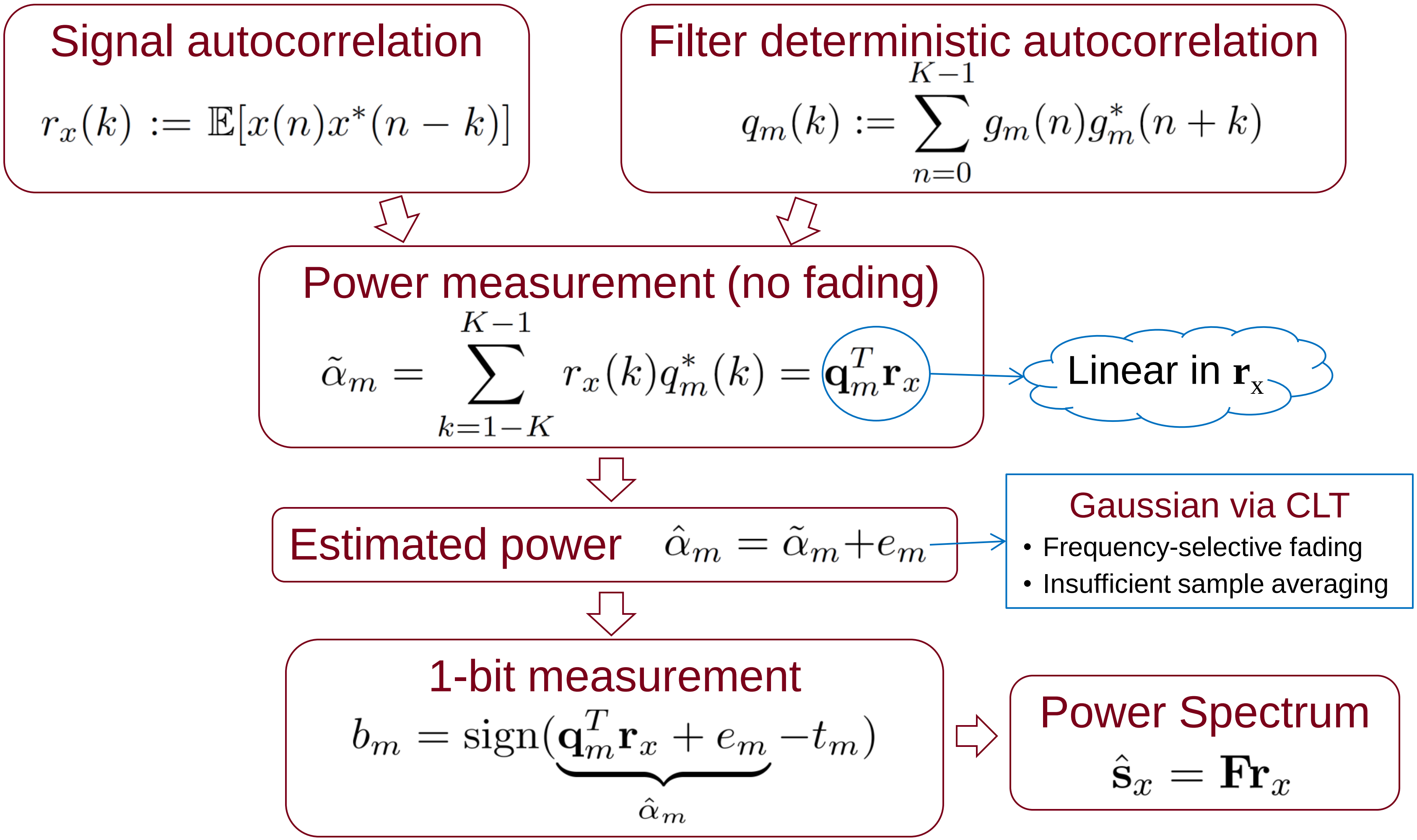
- Random filter output

$$z_m(n) = \sum_{k=0}^{K-1} g_m(k) y_m(n-k) \quad \Rightarrow \quad \alpha_m := \mathbb{E}[|z_m(n)|^2]$$

- Filter output with no fading

$$\tilde{z}_m(n) = \sum_{k=0}^{K-1} g_m(k) x(n-k) \quad \Rightarrow \quad \tilde{\alpha}_m := \mathbb{E}[|\tilde{z}_m(n)|^2]$$

One-Bit Power Measurement



Signal autocorrelation

$$r_x(k) := \mathbb{E}[x(n)x^*(n-k)]$$

Filter deterministic autocorrelation

$$q_m(k) := \sum_{n=0}^{K-1} g_m(n)g_m^*(n+k)$$

Power measurement (no fading)

$$\tilde{\alpha}_m = \sum_{k=1-K}^{K-1} r_x(k)q_m^*(k) = \mathbf{q}_m^T\mathbf{r}_x$$

Linear in $\mathbf{r}_x$

Estimated power $\hat{\alpha}_m = \tilde{\alpha}_m + e_m$

Gaussian via CLT

- Frequency-selective fading
- Insufficient sample averaging

1-bit measurement

$$b_m = \mathrm{sign}(\underbrace{\mathbf{q}_m^T\mathbf{r}_x + e_m}_{\hat{\alpha}_m} - t_m)$$

Power Spectrum

$$\hat{\mathbf{s}}_x = \mathbf{F}\mathbf{r}_x$$

$$\mathbf{r}_x := [r_x(0), \mathrm{Re}\{r_x(1)\}, \ldots, \mathrm{Re}\{r_x(K-1)\}, \mathrm{Im}\{r_x(1)\}, \ldots, \mathrm{Im}\{r_x(K-1)\}]^T$$

$$\mathbf{q}_m := [q_m(0), 2\ \mathrm{Re}\{q_m(1)\}, \ldots, 2\ \mathrm{Re}\{q_m(K-1)\},\ 2\ \mathrm{Im}\{q_m(1)\}, \ldots, 2\ \mathrm{Im}\{q_m(K-1)\}]^T$$

Linear Programming Formulation

- ❑ Assume small $\{e_m\}$: $b_m = \text{sign}(\underbrace{\mathbf{q}_m^T \mathbf{r}_x + e_m}_{\hat{\alpha}_m} - t_m) = \text{sign}(\underbrace{\mathbf{q}_m^T \mathbf{r}_x}_{\tilde{\alpha}_m} - t_m), \ \forall m$
- ❑ Constraints
  - ➢ Received bits $\{b_m\}$: $b_m(\mathbf{q}_m^T \mathbf{r}_x - t_m) \geq 0, \quad m = 1, \ldots, M$
  - ➢ $\mathbf{R}_x = \text{Toeplitz}(\mathbf{r}_x) \succeq 0$ and $\hat{\mathbf{s}}_x = \mathbf{F}\mathbf{r}_x \geq \mathbf{0}$

    Proposition: $\mathbf{F}\mathbf{r}_x \geq 0 \Rightarrow \mathbf{R}_x \succeq 0$
- ❑ Cost function
  - ➢ Minimize total signal power: $E[|x(n)|^2] = r_x(0) = \frac{1}{N_F} \sum_{f=0}^{N_F-1} \hat{s}_x(f) = \frac{1}{N_F} ||\hat{\mathbf{s}}_x||_1$
- ❑ Linear programming

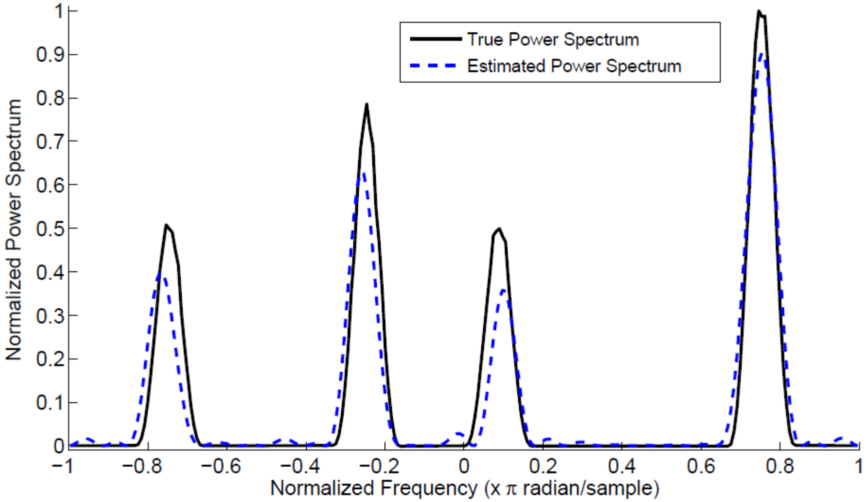
$$\boxed{\begin{array}{ll} \min\limits_{\mathbf{r}_x \in \mathcal{P}} & r_x(0) \\ \text{s.t.} & \mathbf{F}\mathbf{r}_x \geq \mathbf{0}, \quad b_m(\mathbf{q}_m^T \mathbf{r}_x - t_m) \geq 0, \quad m = 1, \ldots, M \end{array}}$$

Spectral estimation from inequalities instead of equalities

O. Mehanna and N. D. Sidiropoulos, “Frugal sensing: wideband power spectrum sensing from few bits,” *Trans. Signal Processing*, May 2013

Simulations

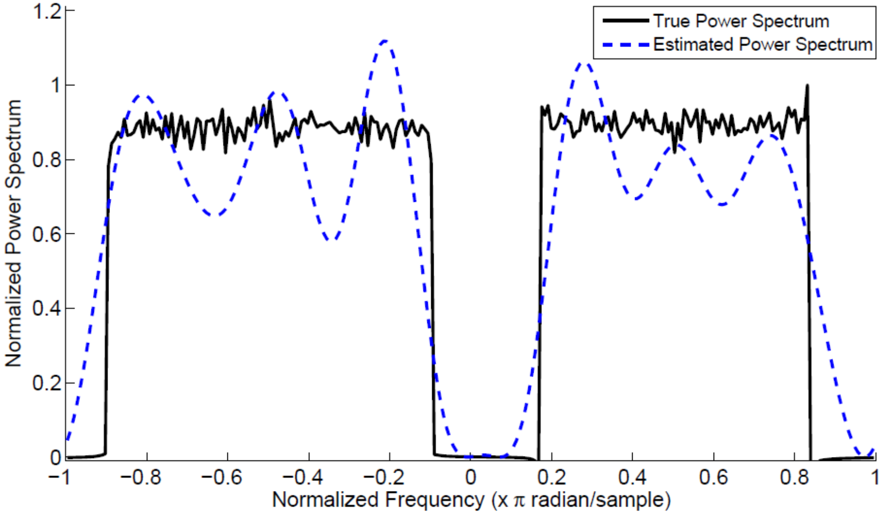
Sparse spectrum



$M=100, K=24, t_m=t$, 30 sensors send $b_m=1$

100 bits equivalent to 3 single precision IEEE floats ($r_x(0)$ and $r_x(1)$)

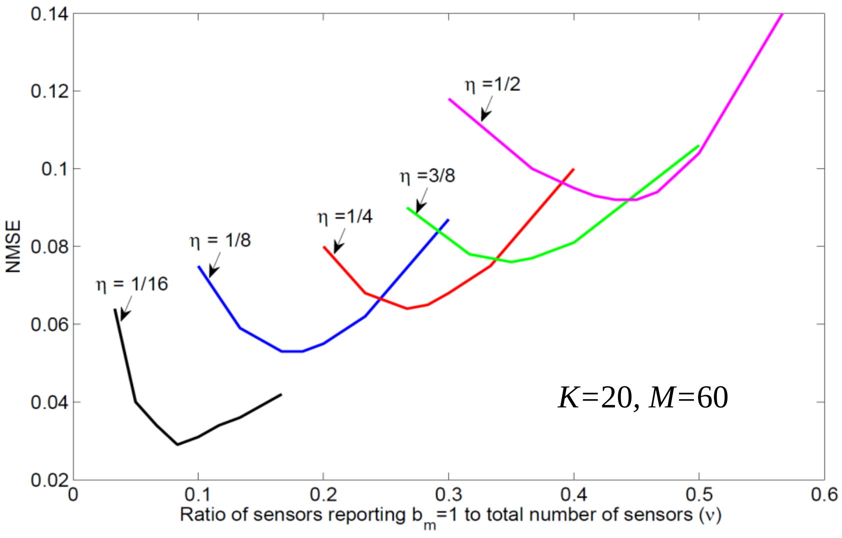
Dense spectrum



$M=100, K=10, t_m=t$, 50 sensors send $b_m=1$

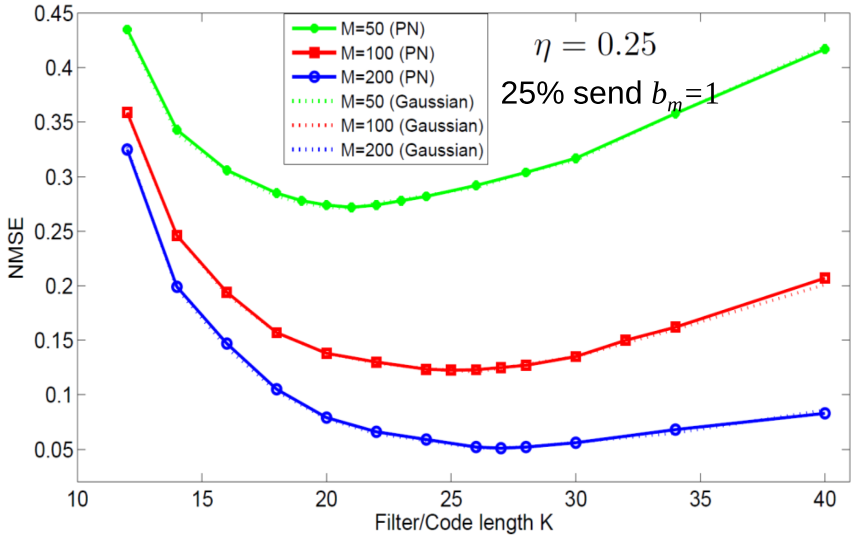
Threshold Selection & Filter Length

Threshold vs. Sparsity



Threshold should be tuned such that fewer sensors are *above threshold* if the power spectrum is more sparse

Filter length vs. Number of sensors



- Small K → smeared power spectrum estimate
- Large K → more unknowns vs. inequality constraints (more under-determined)
- More M → optimal K^* increases
- Binary PN vs. Gaussian

Maximum Likelihood Formulation

i.i.d Gaussian

$$b_m = \text{sign}(\underbrace{\mathbf{q}_m^T \mathbf{r}_x + e_m}_{\hat{\alpha}_m} - t_m) \overset{?}{\neq} \text{sign}(\underbrace{\mathbf{q}_m^T \mathbf{r}_x}_{\tilde{\alpha}_m} - t_m)$$

$$\begin{aligned} \mathbf{b} &:= [b_1, \ldots, b_M]^T \\ \mathcal{M}_+ &:= \{m | b_m = 1\} \\ \mathcal{M}_- &:= \{m | b_m = -1\} \end{aligned}$$

- ❑ Joint PMF

$$\begin{aligned} p[\mathbf{b}|\mathbf{r}_x] &= \prod_{m \in \mathcal{M}_+} p(\mathbf{q}_m^T \mathbf{r}_x + e_m \geq t_m) \prod_{m \in \mathcal{M}_-} p(\mathbf{q}_m^T \mathbf{r}_x + e_m < t_m) \\ &= \prod_{m=1}^{M} Q\left(\frac{-b_m(\mathbf{q}_m^T \mathbf{r}_x - t_m)}{\sigma_m}\right) \end{aligned}$$

- ❑ Constrained ML + Sparsity-inducing penalty

control sparsity

$$\boxed{\begin{aligned} &\max_{\mathbf{r}_x \in \mathcal{P}} \quad \sum_{m=1}^{M} \log Q\left(\frac{-b_m(\mathbf{q}_m^T \mathbf{r}_x - t_m)}{\sigma_m}\right) - \lambda\, r_x(0) \\ &\text{s.t.} \quad \mathbf{F}\mathbf{r}_x \geq \mathbf{0} \end{aligned}}$$

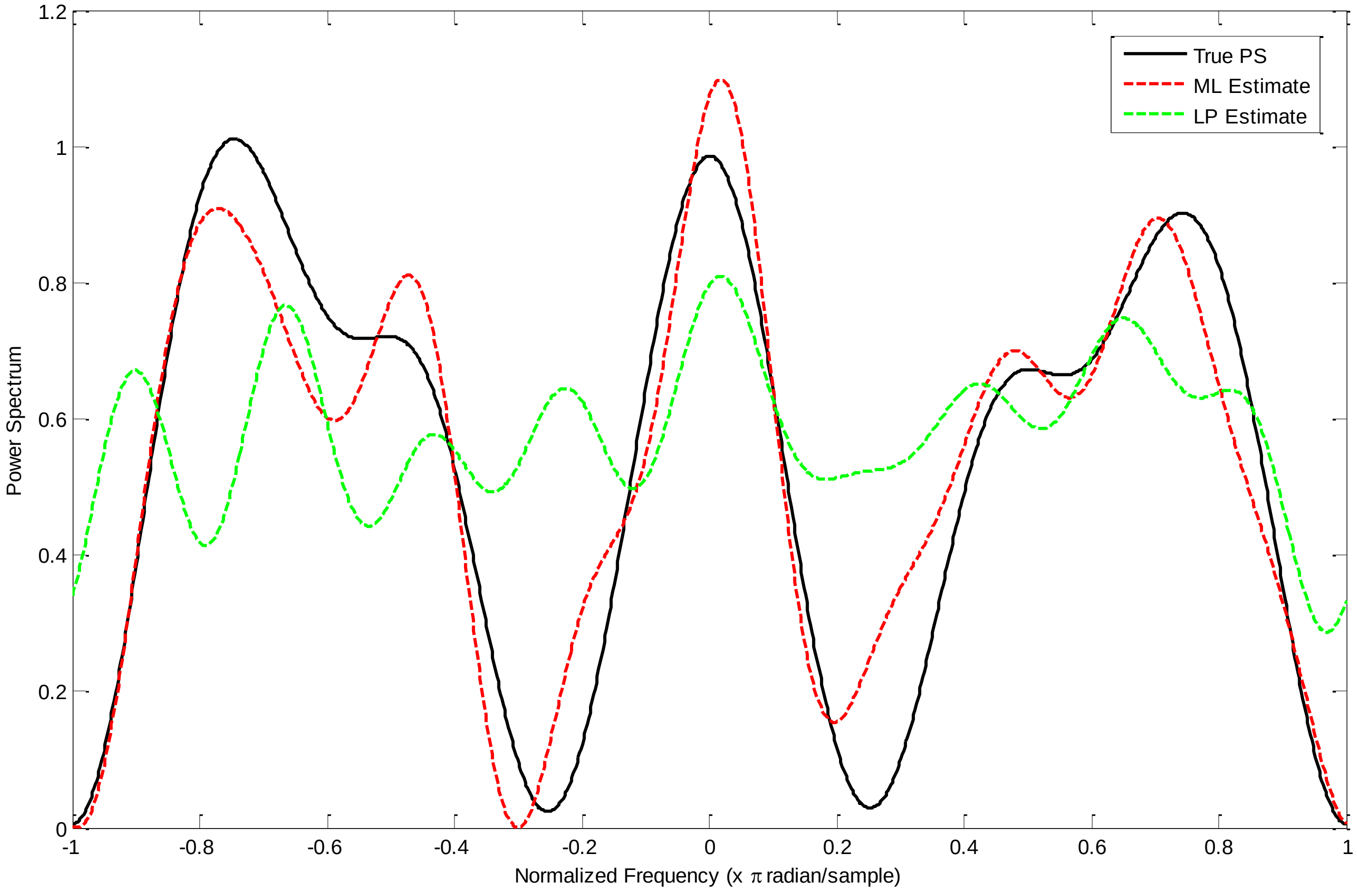
Convex

Consistent

O. Mehanna and N. D. Sidiropoulos, "Maximum likelihood passive and active sensing of wideband power spectra from few bits," *TSP*, in review

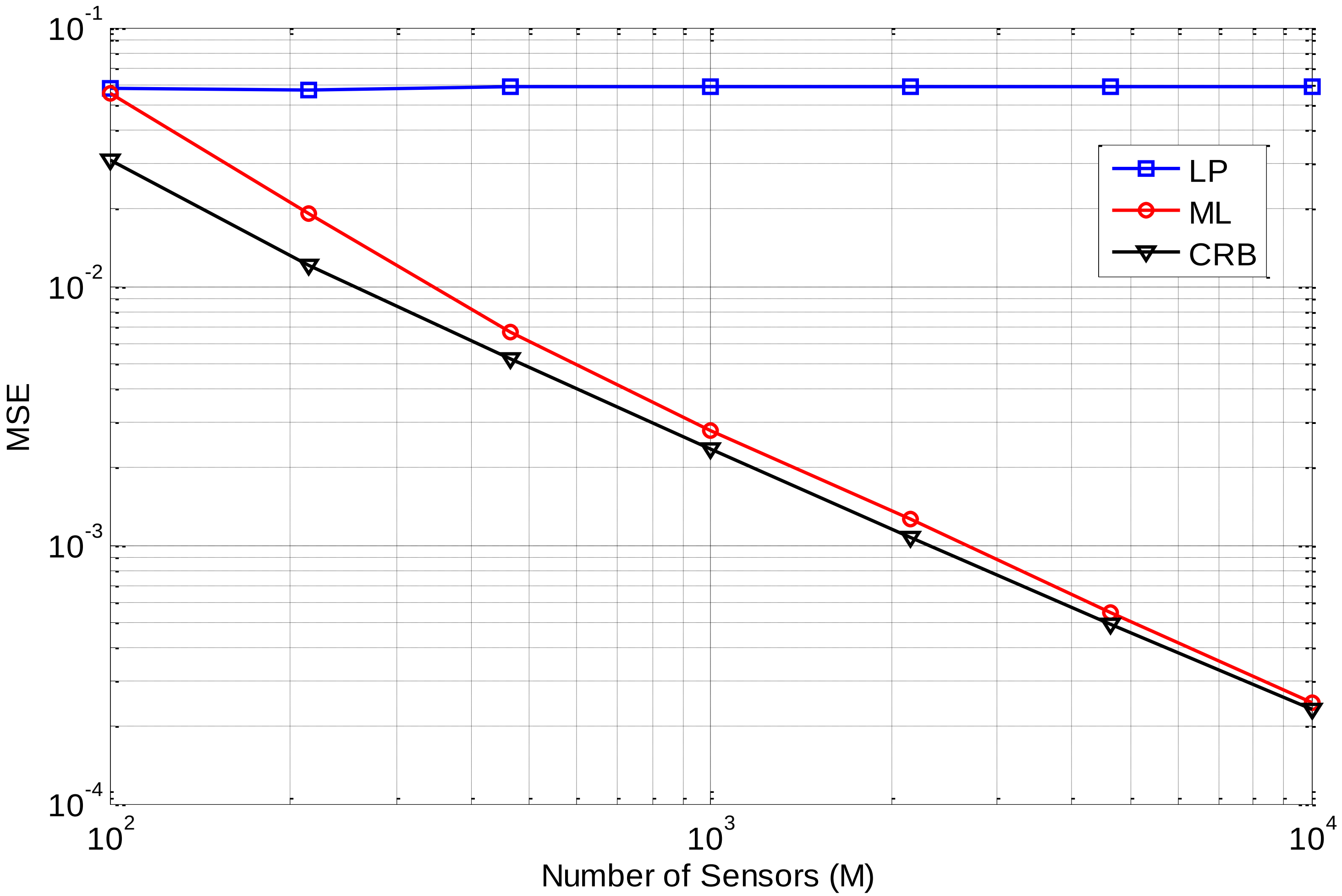
Example: ML vs. LP

M=150, K=10, t_m=t, 50 sensors send b_m=1,
random errors flipped 24 sensor measurement bits (16%)



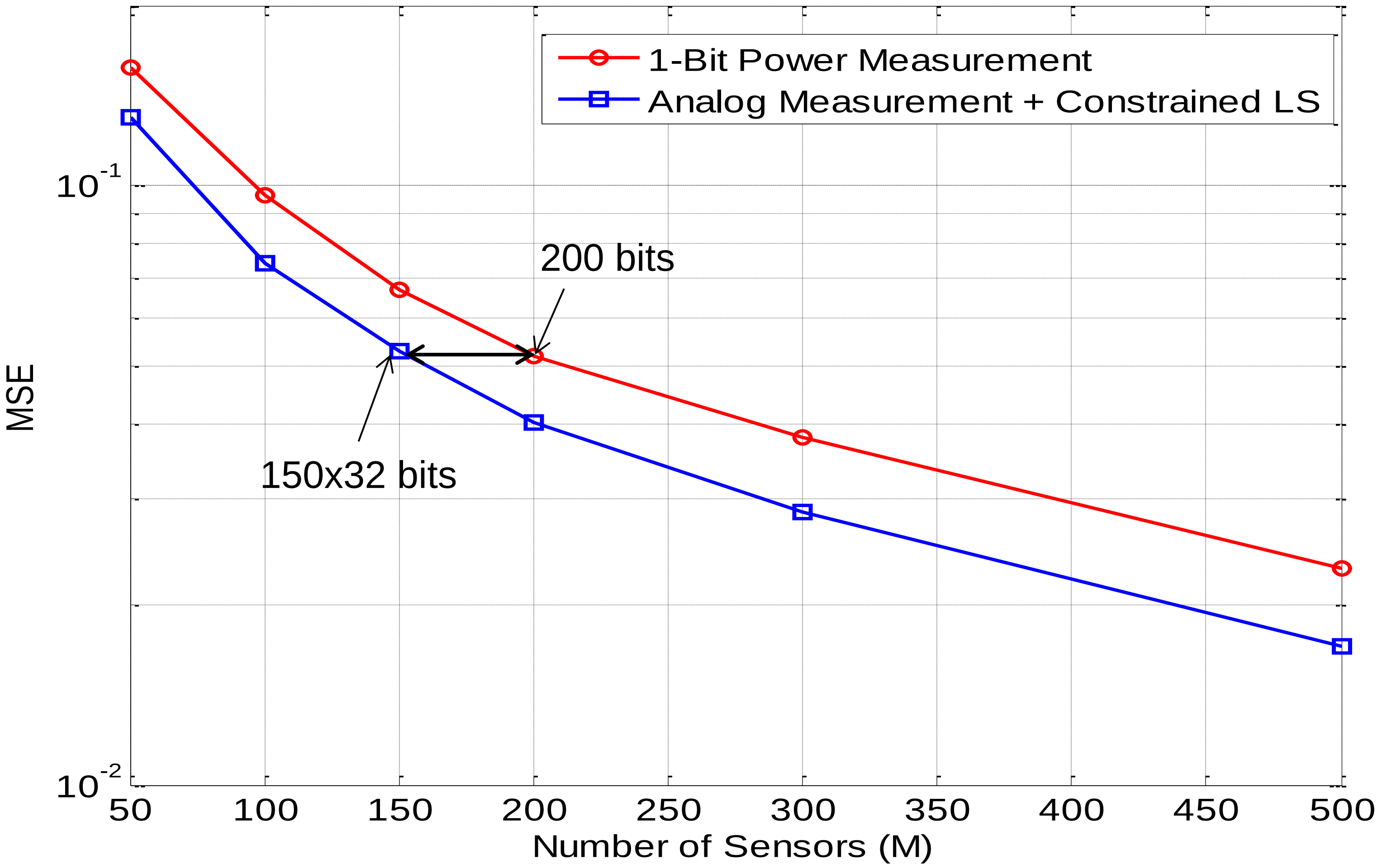
MSE vs. CRB

Random errors flipped 17% of sensor measurement bits on average



Analog vs. 1-Bit Quantization

Rayleigh fading: random errors flipped 30% of sensor measurement bits on average



Active Sensing

(Adaptive Thresholding)

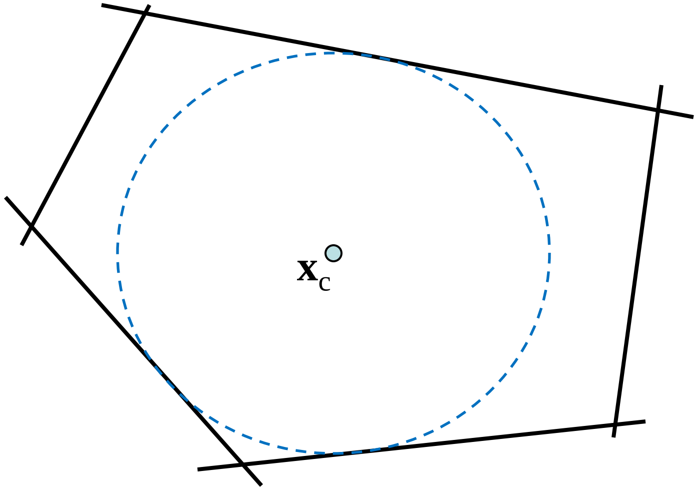
Threshold Selection

- Satisfactory estimation quality with fixed $t_m = t$ for all sensors
- What if $\{t_m\}$ actively adapted online and communicated to sensors through downlink channel?
  - Significant payoff in terms of sensing accuracy
  - At the cost of higher complexity and communication overhead
- Assume accurate power measurements ($\mathrm{sign}(\hat{\alpha}_m - t_m) = \mathrm{sign}(\tilde{\alpha}_m - t_m), \forall m$)
  - Receiving the M bits:

$$\mathcal{P}_M = \mathcal{P}_0 \cap \{\mathbf{x} \mid b_m(\mathbf{q}_m^T \mathbf{x} - t_m) \geq 0,\ \ m = 1, \ldots, M\}$$

  - Volume of feasible region gives a measure of uncertainty about $\mathbf{r}_x$
- Objective: adaptively select $\{t_m\}$ to ensure $\mathcal{P}_M$ is as small as possible

Chebyshev Center (CC)



- The CC of $\mathcal{C} := \{\mathbf{x} \mid \mathbf{a}_i^T\mathbf{x} \leq c_i,\ i = 1, \ldots, L\}$ found by solving the LP

$$\boxed{\begin{aligned} \max_{R \geq 0, \mathbf{x}} \quad & R \\ \text{s.t. :} \quad & \mathbf{a}_i^T\mathbf{x} + R||\mathbf{a}_i||_2 \leq c_i,\ i = 1, \ldots, L \end{aligned}}$$

Adaptive Thresholding Algorithm

Given $\mathcal{P}_0$, its CC $\mathbf{x}_c^{(0)}$, and $\{\mathbf{q}_m\}_{m=1}^{M}$

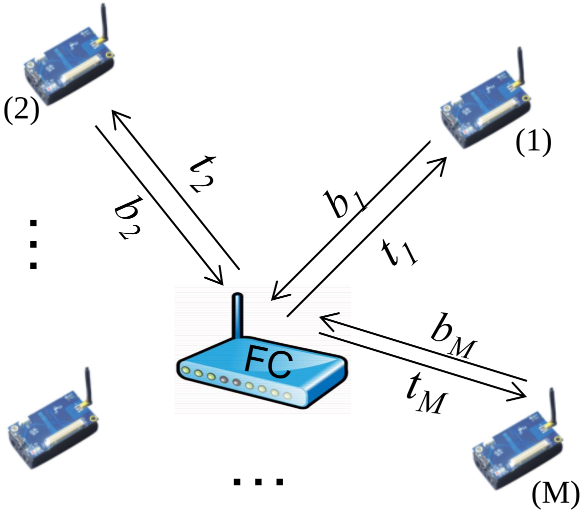
For each time-slot / sensor $m=1,\ldots,M$, do

1. Set $t_m = \mathbf{q}_m^T\mathbf{x}_c^{(m-1)}$, send it to senor m
2. Upon receiving b_m update:

$$\mathcal{P}_m := \begin{cases} \mathcal{P}_{m-1} \cap \{\mathbf{x} \mid \mathbf{q}_m^T\mathbf{x} \geq t_m\} & \text{if } b_m = 1 \\ \mathcal{P}_{m-1} \cap \{\mathbf{x} \mid \mathbf{q}_m^T\mathbf{x} < t_m\} & \text{if } b_m = -1 \end{cases}$$

3. Compute the CC $\mathbf{x}_c^{(m)}$ of $\mathcal{P}_m$

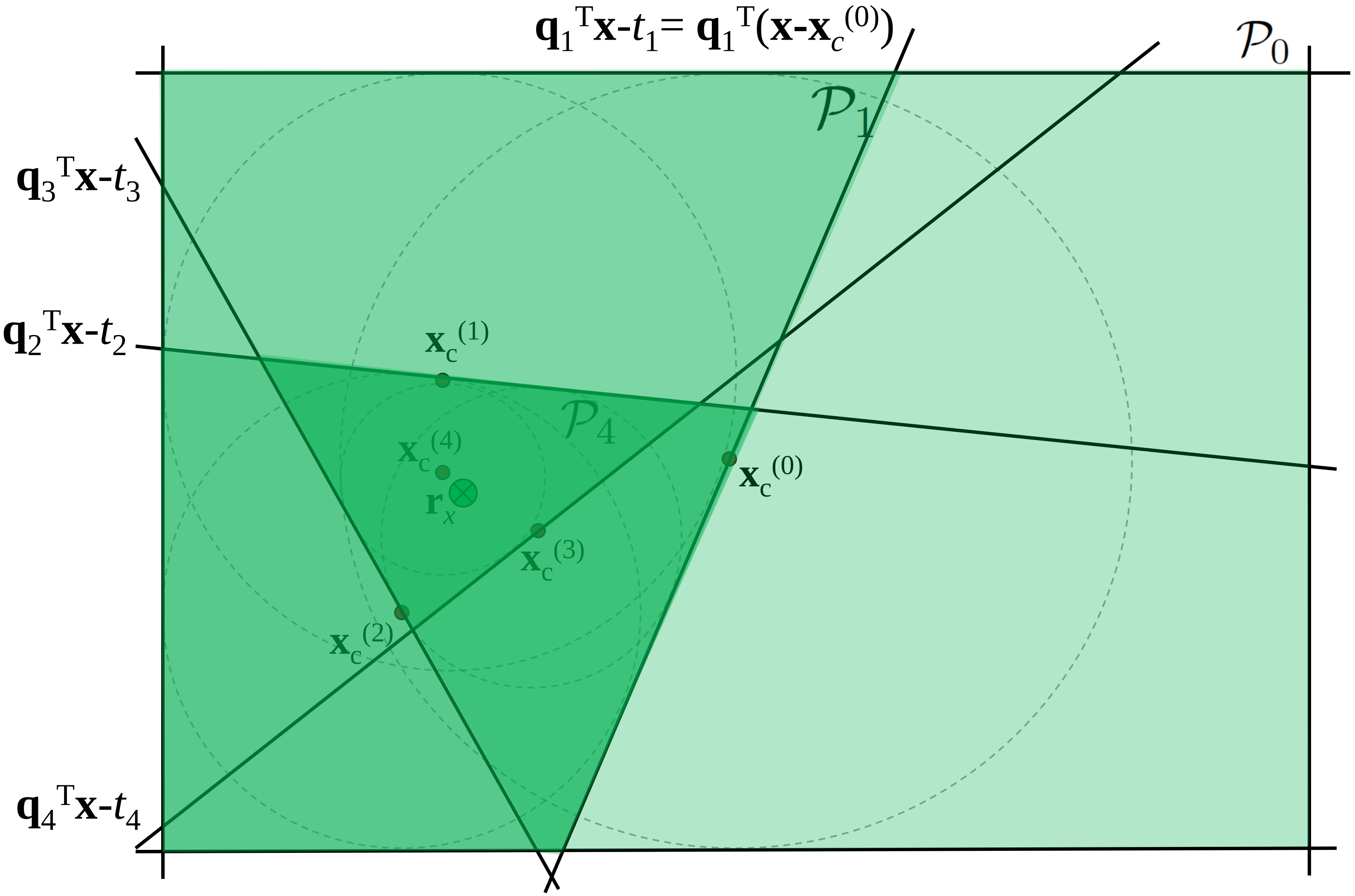
$$\boxed{\hat{\mathbf{r}}_x = \mathbf{x}_c^{(M)}}$$



$\mathbf{x}_c^{(M)}$ converges linearly to $\mathbf{r}_x$ as $M \rightarrow \infty$

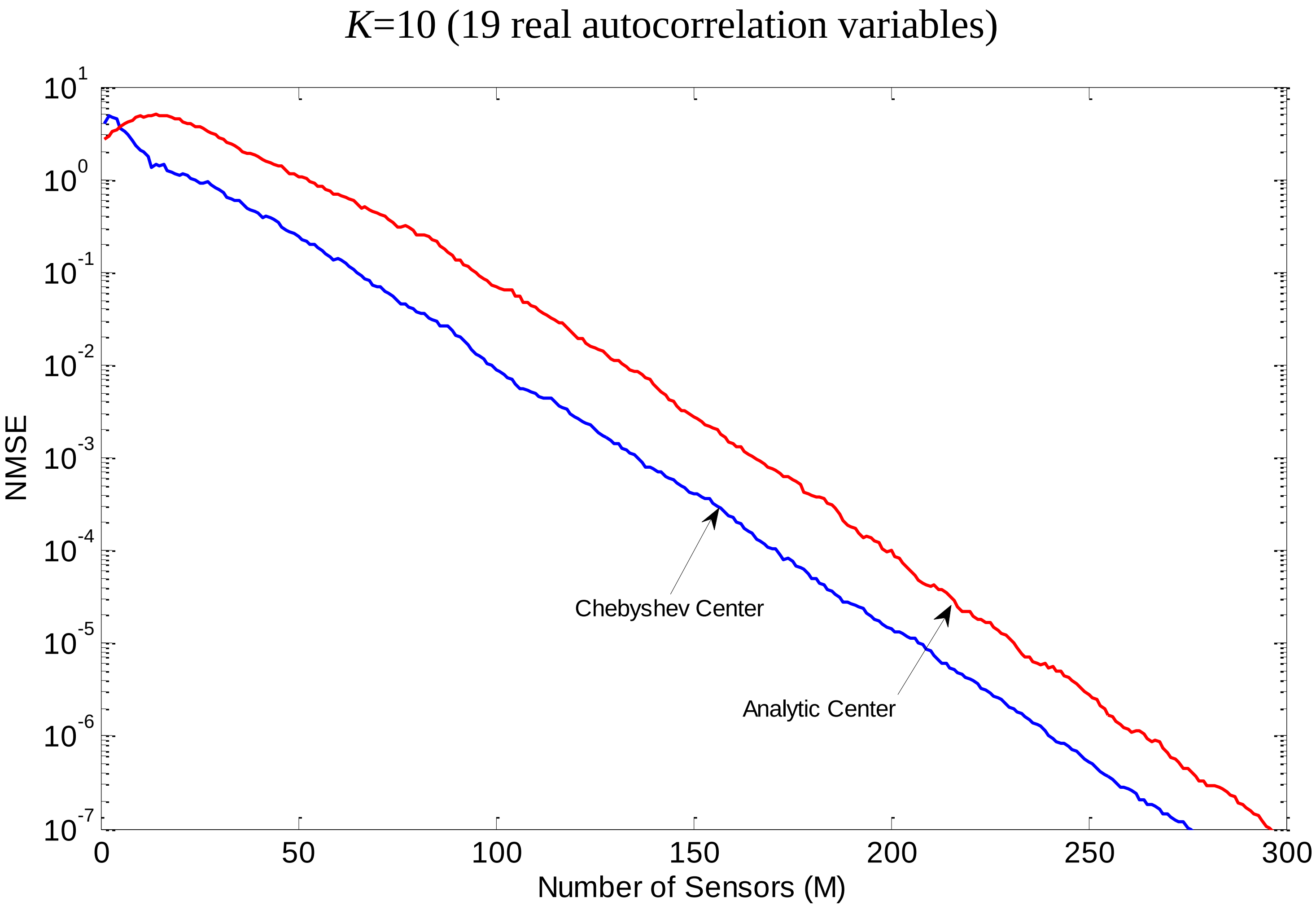
O. Mehanna and N. D. Sidiropoulos, “Adaptive thresholding for distributed power spectrum sensing,” *ICASSP 2013*

2-D Example



Significant portion of the feasible region is cut-off after each iteration

Active Sensing Performance



Active Sensing with Gaussian Errors

- Bit-flips due to errors prevent convergence of previous scheme
  - Received measurement bit does not imply inequality constraint, does not decrease the feasible region

- Instead of CC, use ML estimate

$$\mathbf{r}_x^{(m)} = \arg \max_{\mathbf{r}_x \in \mathcal{P}} \sum_{i=1}^{m} \log Q\left(\frac{-b_i(\mathbf{q}_i^T \mathbf{r}_x - t_i)}{\sigma_i}\right)$$

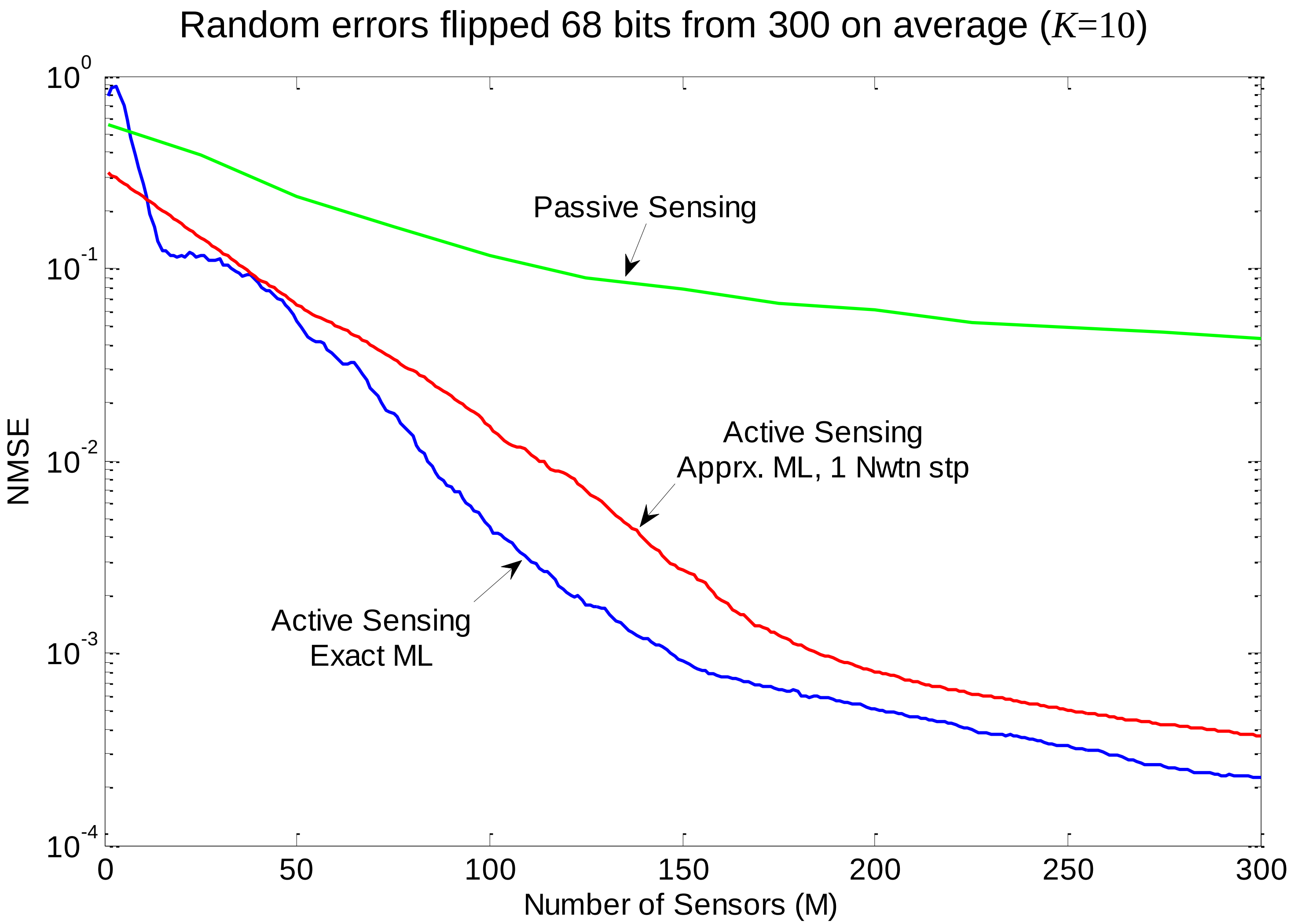
  - Set $t_{m+1} = \mathbf{q}_{m+1}^T \mathbf{r}_x^{(m)}$
  - CRB minimized with $t_m^* = \mathbf{q}_m^T \mathbf{r}_x$

- Low-complexity (approximate ML)

$$\mathbf{r}_x^{(m)} = \mathbf{r}_x^{(m-1)} - \left(\nabla^2 \Gamma_m(\mathbf{r}_x^{(m-1)})\right)^{-1} \nabla \Gamma_m(\mathbf{r}_x^{(m-1)})$$

$$\Gamma_m(\mathbf{x}) := -\sum_{i=1}^{m} \log Q\left(\frac{-b_i(\mathbf{q}_i^T \mathbf{x} - t_i)}{\sigma_i}\right)$$

Performance with Errors



Parametric Sensing

Parametric Frugal Sensing

- Assume primary WSS signal admits an MA(*q*) representation

$$x(n) = \sum_{k=0}^{q} h(k) w(n-k)$$

MA parameters Complex WGN $\sim \mathcal{CN}(0,1)$

- MA Signal Autocorrelation:

$$r_x(k) = \begin{cases} \sum_{i=0}^{q-|k|} h^*(i) h(i+|k|) & : |k| \leq q \\ 0 & : |k| > q \end{cases}$$

$$= \begin{cases} \mathbf{h}^H \boldsymbol{\Theta}_k^{(q+1)} \mathbf{h} & : |k| \leq q \\ 0 & : |k| > q \end{cases}$$

- MA power spectrum: $S_x(e^{j\omega}) = |\sum_{n=0}^{q} h(n) e^{-j\omega n}|^2$

- Parametric approach yields more parsimonious model for power spectrum

QCQP Formulation

- For small $\{e_m\}$: $\tilde{\alpha}_m = \mathbb{E}[|\tilde{z}_m|^2] = \mathbb{E}[|\mathbf{g}_m^H \mathbf{x}|^2] = \boxed{\mathbf{g}_m^H \mathbf{R}_x \mathbf{g}_m} \longleftarrow$ Linear in the auto-correlation

- Assuming a postulated model order *p*

$$
\begin{aligned}
\mathbf{g}_m^H \mathbf{R}_x \mathbf{g}_m &= \mathbf{g}_m^H \left(r(0)\boldsymbol{\Theta}_0^K + \sum_{k=1}^{\min(K-1,p)} \left(r(k)\boldsymbol{\Theta}_k^K + r^*(k)\boldsymbol{\Theta}_{-k}^K \right) \right) \mathbf{g}_m \\
&= \underbrace{\mathbf{g}_m^H \boldsymbol{\Theta}_0^K \mathbf{g}_m}_{c_{m,0}} r(0) + \sum_{k=1}^{min(K-1,p)} \left(\underbrace{\mathbf{g}_m^H \boldsymbol{\Theta}_k^K \mathbf{g}_m}_{c_{m,k}} r(k) + \underbrace{\mathbf{g}_m^H \boldsymbol{\Theta}_{-k}^K \mathbf{g}_m}_{c_{m,-k}} r^*(k) \right) \\
&= c_{m,0} r(0) + \sum_{k=1}^{\min(K-1,p)} \left(c_{m,k} r(k) + c_{m,-k} r^*(k) \right) \\
&= \mathbf{h}^H \left(c_{m,0} \boldsymbol{\Theta}_0^{(p+1)} + \sum_{k=1}^{min(K-1,p)} \left(c_{m,k} \boldsymbol{\Theta}_k^{(p+1)} + c_{m,-k} \boldsymbol{\Theta}_{-k}^{(p+1)} \right) \right) \mathbf{h} \\
&= \boxed{\mathbf{h}^H \mathbf{C}_m \mathbf{h}} \longrightarrow \text{Quadratic in the MA parameters}
\end{aligned}
$$

QCQP Formulation

- Constraints
  - Assume single threshold: $t_m = t$
  - Define sets $\mathcal{M}_a := \{m : b_m = 1\}, \mathcal{M}_b := \{m : b_m = -1\}, |\mathcal{M}_a| + |\mathcal{M}_b| = M$
  - Received bits $\{b_m\}$: $b_m = 1 \implies \mathbf{h}^H \mathbf{C}_m \mathbf{h} \geq t, \ \forall m \in \mathcal{M}_a$

    $b_m = -1 \implies \mathbf{h}^H \mathbf{C}_m \mathbf{h} < t, \ \forall m \in \mathcal{M}_b$

- Cost function
  - Minimize total signal power: $\mathbb{E}[|x(n)|^2] = r_x(0) = \mathbf{h}^H \mathbf{h} = \|\mathbf{h}\|_2^2$

- Quadratically Constrained Quadratic Programming (QCQP)

$$\boxed{\begin{aligned} (P) \quad \min_{\mathbf{h} \in \mathbb{C}^{p+1}} \quad & \|\mathbf{h}\|_2^2 \\ \text{s.t.} \quad & \mathbf{h}^H \mathbf{C}_m \mathbf{h} \geq t, \ m \in \mathcal{M}_a \\ & \mathbf{h}^H \mathbf{C}_m \mathbf{h} < t, \ m \in \mathcal{M}_b \end{aligned}}$$

Non-convex problem, known to be NP-Hard

A. Konar, N. D. Sidiropoulos, and O. Mehanna “Parametric Frugal Sensing of Power Spectra for Moving Average Models” *TSP (submitted)*

Semidefinite Relaxation Approach

- Equivalent reformulation of (P)

$$\boxed{\begin{array}{cl} \underset{\mathbf{H}\in\mathbb{C}^{p+1\times p+1}}{\text{min.}} & \text{Trace}(\mathbf{H}) \\ \text{s.t.} & \text{Trace}(\mathbf{C}_m\mathbf{H}) \geq t,\ m \in \mathcal{M}_a \\ & \text{Trace}(\mathbf{C}_m\mathbf{H}) < t,\ m \in \mathcal{M}_b \\ & \mathbf{H} \succeq \mathbf{0}, \\ & \text{rank}(\mathbf{H}) = 1 \end{array}}$$

- Relaxed semidefinite programming (SDP) problem obtained by dropping rank constraints

$$\boxed{\begin{array}{cl} \underset{\mathbf{H}\in\mathbb{C}^{p+1\times p+1}}{\text{min.}} & \text{Trace}(\mathbf{H}) \\ \text{s.t.} & \text{Trace}(\mathbf{C}_m\mathbf{H}) \geq t,\ m \in \mathcal{M}_a \\ & \text{Trace}(\mathbf{C}_m\mathbf{H}) < t,\ m \in \mathcal{M}_b \\ & \mathbf{H} \succeq \mathbf{0} \end{array}}$$

Randomization Algorithm

- If SDP solution is rank 1, then global optimum achieved

- Randomization Approach
  - Scale prinicipal component of SDP solution to be feasible for (P)
  - Employ *Gaussian Randomization* to obtain feasible solution
  - If randomization fails to obtain a feasible solution,
    - Scale principal component/use Gaussian Randomization to obtain feasible solution for $\mathcal{M}_a$ only
    - Justification: $\mathcal{M}_a$ is the activity detection set, MVDR interpretation

Successive Convex Approximation Approach

- Linearize $f_m(\mathbf{h}) = \mathbf{h}^H \mathbf{C}_m \mathbf{h}$ about point $\mathbf{p}$ for $\mathcal{M}_a$ to obtain lower bound $F_m(\mathbf{h}, \mathbf{p}) = \mathcal{R}\mathbf{e}\{\mathbf{a}_m^H \mathbf{h}\} - b_m$ where $\mathbf{a}_m = 2\mathbf{C}_m \mathbf{p},\ b_m = \mathbf{p}^H \mathbf{C}_m \mathbf{p}$

- Proposition: Seek to "solve" (P) by solving the sequence of convex problems

$$\boxed{\begin{array}{lll} (P_k) & \underset{\mathbf{h} \in \mathbb{C}^{p+1}}{\min.} & \|\mathbf{h}\|_2^2 \\ & \text{s.t.} & F_m(\mathbf{h}, \mathbf{p}_k) \geq t, m \in \mathcal{M}_a \\ & & \mathbf{h}^H \mathbf{C}_m \mathbf{h} < t, m \in \mathcal{M}_b \end{array}}$$

- If (P) is feasible and $\mathbf{p}_0$ is a feasible starting point, then, it can be shown that the sequence of solutions generated has monotonically non-increasing cost, and converges to a KKT point. [Beck-Ben Tal-Tetruashvili '10]

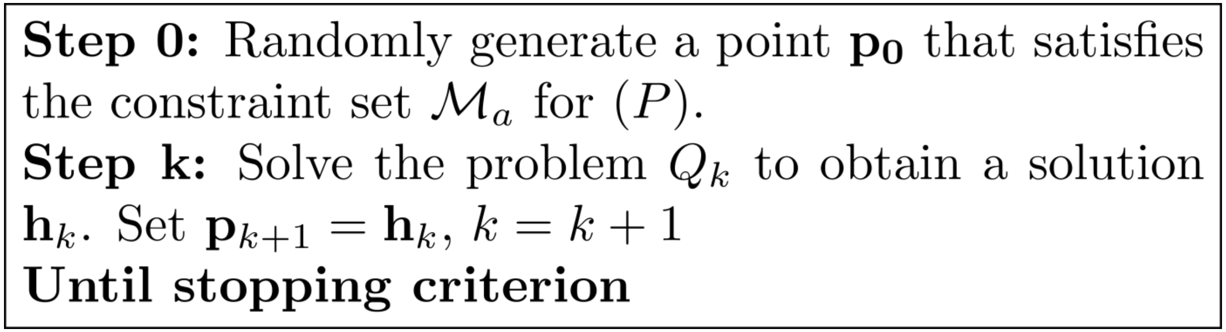
SOCP Formulation with slack variables

- Drawbacks: Obtaining a feasible starting point $\mathbf{p}_0$ non-trivial.
- Alternative: Choose $\mathbf{p}_0$ to be feasible for $\mathcal{M}_a$ only
- Issues: (P_k) maybe infeasible as a result of computing restriction of $\mathcal{M}_a$ about $\mathbf{p}_0$
- Fixes:
  - Add positive slack variables $\{s_i\}_{i=1}^{M_b}$ to convex constraints
  - Impose a weighted penalty on the sum-of-slacks
  - Scale $F_m(\mathbf{h}, \mathbf{p})$ until it becomes tangent to the hyper-ellipse $\mathbf{h}^H \mathbf{C}_m \mathbf{h} = t, \forall\ m \in \mathcal{M}_a$
- Overall, we obtain the following problem

$$\boxed{\begin{array}{lll} (Q_k) & \underset{\mathbf{h}\in\mathbb{C}^{p+1}, \mathbf{s}\in\mathbb{R}^{M_b}}{\text{min.}} & \|\mathbf{h}\|_2^2 + \lambda \sum_{i=1}^{M_b} s_i \\ & \text{s.t.} & F_m(\mathbf{h}, \alpha_m \mathbf{p}_k) \geq t,\ m \in \mathcal{M}_a \\ & & \mathbf{h}^H \mathbf{C}_m \mathbf{h} < t + s_m,\ m \in \mathcal{M}_b \\ & & \mathbf{s} \succeq \mathbf{0} \end{array}}$$

Can be formulated as a SOCP problem

Feasible Point Pursuit Algorithm



Step 0: Randomly generate a point $\mathbf{p_0}$ that satisfies the constraint set $\mathcal{M}_a$ for (P).

Step k: Solve the problem Q_k to obtain a solution $\mathbf{h}_k$. Set $\mathbf{p}_{k+1} = \mathbf{h}_k$, $k = k + 1$

Until stopping criterion

- Cost function is monotonically non-increasing in k
- Additionally, $\|\mathbf{h}_{k+1}\|_2^2 \leq \|\mathbf{h}_k\|_2^2$, $\|\mathbf{s}_{k+1}\|_1 \leq \|\mathbf{s}_k\|_1$
- Furthermore, $\mathbf{s}_k \longrightarrow \mathbf{0}$ in a finite number of iterations in many cases i.e., a feasible point is obtained.
- Stop if feasibility achieved in ≤ 30 iterations. Otherwise re-initialize from a different starting point. (Maximum of 5)

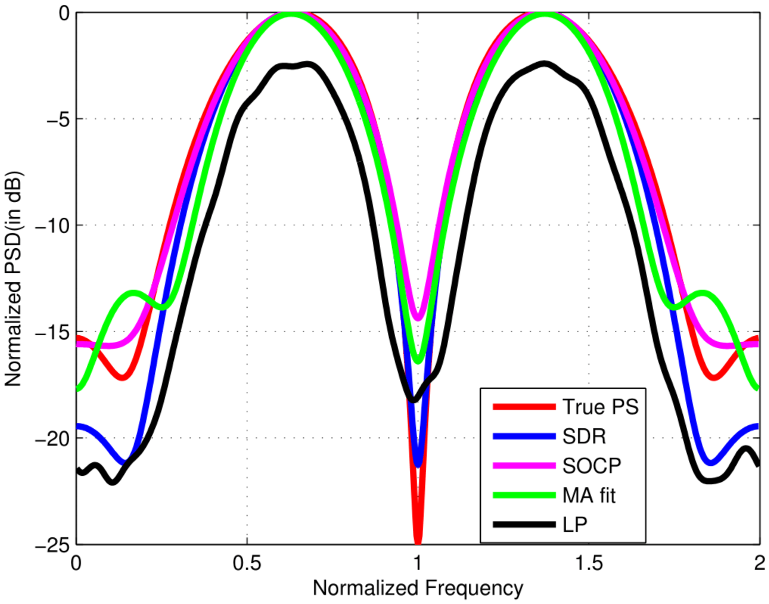
MA Model Fitting Approach

- Fit an MA model of desired order to autocorrelation sequence estimate [Stoica-Moses'05]
  - Use autocorrelation estimate $\{\hat{r}_x(k)\}_{k=-(K-1)}^{K-1}$ returned by LP formulation as starting point
  - Seek to solve the problem $\boxed{\min. \quad \frac{1}{2\pi}\int_{-\pi}^{\pi}[\hat{S}_x(e^{j\omega}) - S_x(e^{j\omega})]^2 \mathrm{d}\omega}$

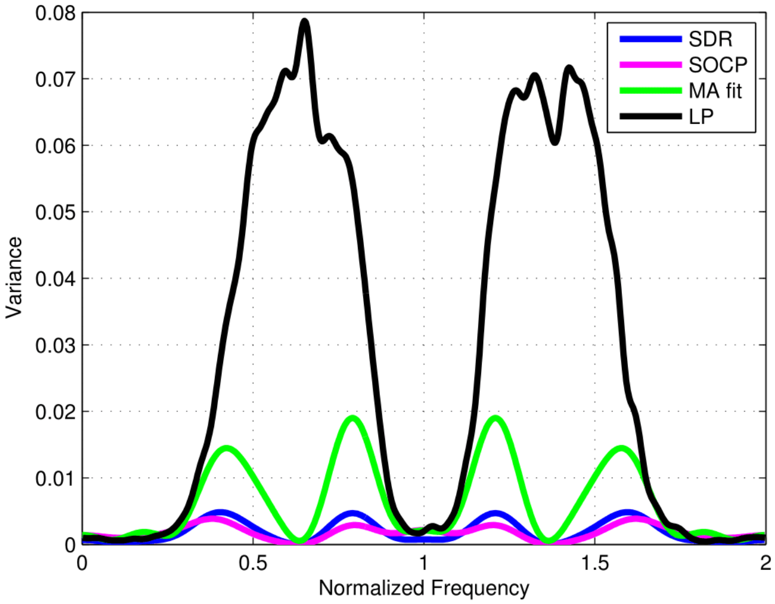
    where $\hat{S}_x(e^{j\omega}) = \sum_{k=-(K-1)}^{K-1} \hat{r}_x(k)e^{-j\omega k}$ and $S_x(e^{j\omega}) = \sum_{k=-p}^{p} r_x(k)e^{-j\omega k}$
  - Can be formulated as a Semidefinite Quadratic Linear Programming (SQLP) problem in $r_x(k)$
  - Take DTFT of $r_x(k)$ to obtain spectral estimate

Simulations

Mean Normalized Spectra



Variance

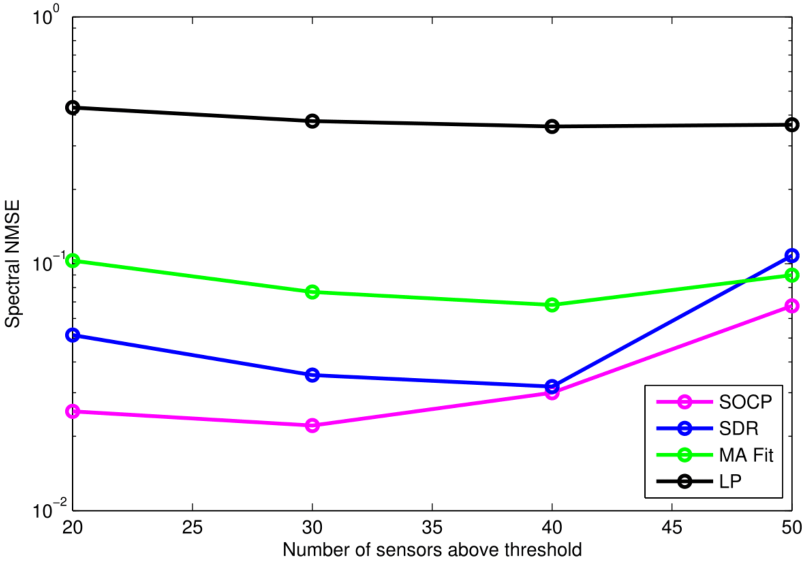


Real MA(5) model, M=100, K=24, t_m=t, 30 sensors send b_m=1, true model order known, 500 MC trials

SDR fails in 99.8 % of trials. FPP – SCA successful in 100 % of trials (avg. 2.3 iterations)

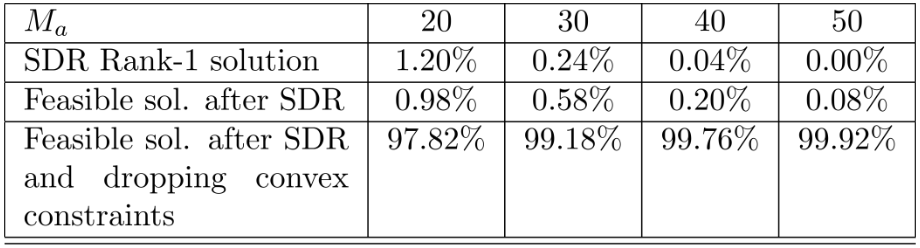
Mean Spectral NMSE

$M=100$, $K=34$, $t_m=t$, true model order known, 100 MC trials for each of 50 MA models



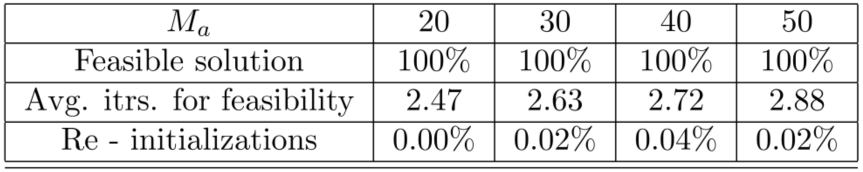
MA(9) model

Parametric methods exhibit superior performance



M_a	20	30	40	50
SDR Rank-1 solution	1.20%	0.24%	0.04%	0.00%
Feasible sol. after SDR	0.98%	0.58%	0.20%	0.08%
Feasible sol. after SDR and dropping convex constraints	97.82%	99.18%	99.76%	99.92%

Table 1: Results using the SDR approach.

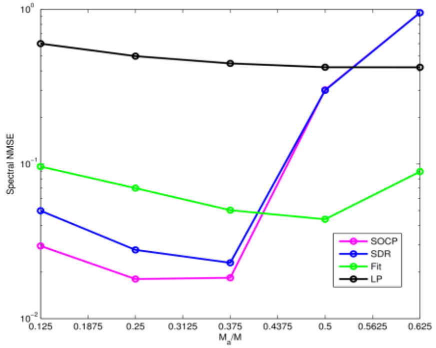


M_a	20	30	40	50
Feasible solution	100%	100%	100%	100%
Avg. itrs. for feasibility	2.47	2.63	2.72	2.88
Re - initializations	0.00%	0.02%	0.04%	0.02%

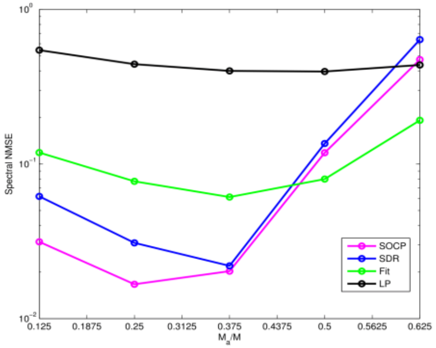
Table 2: Results using the iterative SOCP approach.

FPP – SCA algorithm more successful in obtaining feasible solution

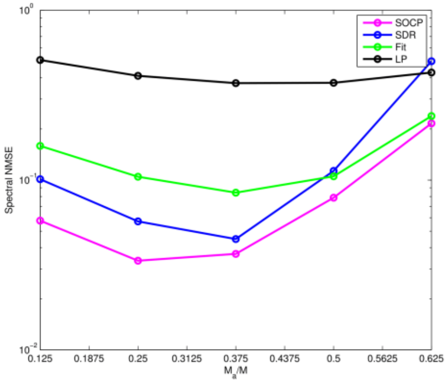
Threshold Selection



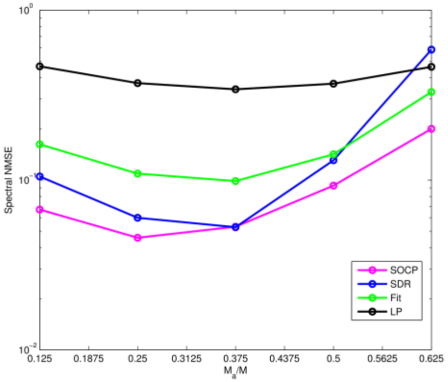
MA(3) model



MA(6) model



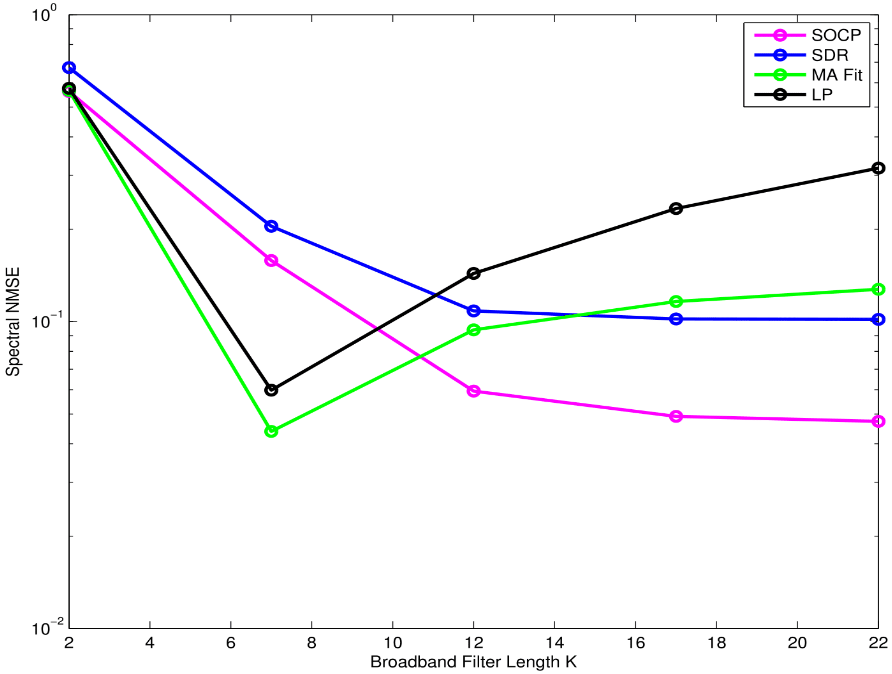
MA(9) model



MA(12) model

- *M=80, K=30, t_m=t, true model order known, 100 MC trials for each of 50 MA models*
- Optimal choice of threshold corresponds to 25-35% of sensors transmitting b_m=1

Broadband Filter Length K

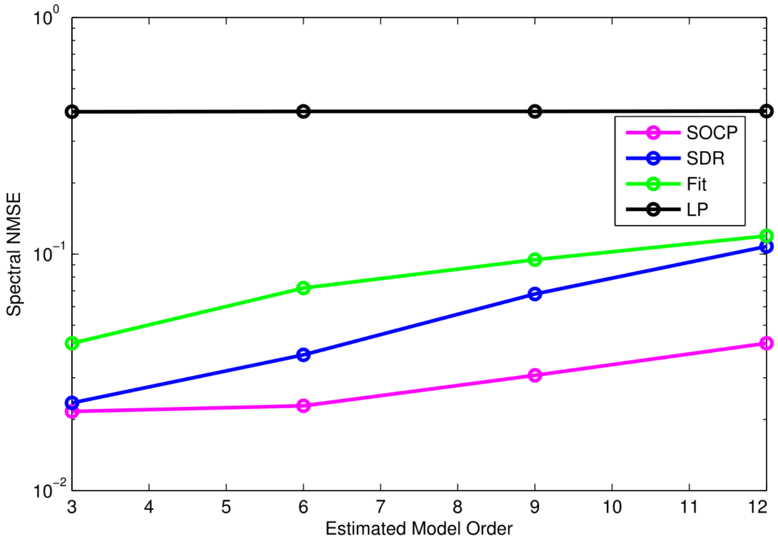


MA(6) model

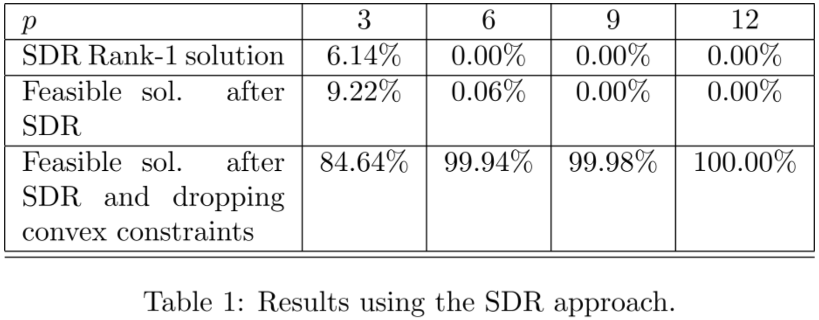
- $M=100$, $t_m=t$, 20 sensors send $b_m=1$, true model order known
- K should be set greater than equal to $q+1$ for correct parametrization of MA model
- For $K = 7$, LP formulation followed by MA model fitting yields best results
- For larger values of K, LP formulation becomes more underdetermined, modelling mismatch increases, hence performance degrades.
- Parametric methods exhibit improved performance for large K

Postulated Model Order p

M=100, K=24, t_m=t, only upper bound on true model order available, 40 sensors send b_m=1, 100 MC trials for each of 50 MA models

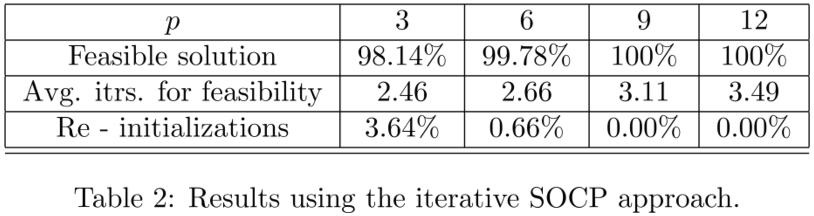


MA(3) model



p	3	6	9	12
SDR Rank-1 solution	6.14%	0.00%	0.00%	0.00%
Feasible sol. after SDR	9.22%	0.06%	0.00%	0.00%
Feasible sol. after SDR and dropping convex constraints	84.64%	99.94%	99.98%	100.00%

Table 1: Results using the SDR approach.



p	3	6	9	12
Feasible solution	98.14%	99.78%	100%	100%
Avg. itrs. for feasibility	2.46	2.66	3.11	3.49
Re - initializations	3.64%	0.66%	0.00%	0.00%

Table 2: Results using the iterative SOCP approach.

FPP – SCA algorithm more robust to model order estimation

FPP – SCA algorithm more successful in obtaining feasible solution

Take-home

- Frugal sensing
  - Applicable for crowdsourcing spectrum sensing using smart phones
  - Adequate wideband power spectrum sensing from few bits
    - Spectral estimation from inequalities instead of equalities
    - LP formulation
    - ML formulation exploits Gaussian errors, robust to bit-flips
  - Active sensing (adaptive thresholding)
    - Fast convergence using adapted threshold information
  - Parametric frugal sensing for MA models
- Ongoing work: AR, ARMA FS; *active* MA FS
- Frugal channel tracking (didn't have time to cover)
  - Results pave the way for using massive MIMO in FDD mode

References

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  - A. Konar, N. D. Sidiropoulos, and O. Mehanna, “Parametric Frugal Sensing of Power Spectra for Moving Average Models”, *IEEE Transactions on Signal Processing*, submitted
  - O. Mehanna and N. D. Sidiropoulos, “Channel tracking and transmit beamforming with frugal feedback,” *IEEE Transactions on Signal Processing*, submitted
  - O. Mehanna and N. D. Sidiropoulos, “Maximum likelihood passive and active sensing of wideband power spectra from few bits,” *IEEE Transactions on Signal Processing*, in review
  - O. Mehanna and N. D. Sidiropoulos, “Frugal sensing: wideband power spectrum sensing from few bits,” *IEEE Transactions on Signal Processing*, vol. 61, no. 10, pp. 2693–2703, May 2013

- ☐ Conference
  - O. Mehanna, N. D. Sidiropoulos, and E. Tsakonas “Model-based power spectrum sensing from a few bits,” *Proc. of 21st European Signal Processing Conference*, Marrakech, Morroco, Sept. 9-13, 2013
  - O. Mehanna, N. D. Sidiropoulos, and E. Tsakonas, “Line spectrum estimation from broadband power detection bits,” *Proc. of the 14th Workshop on Signal Processing Advances in Wireless Communications*, Darmstadt, Germany, June 16-19, 2013
  - O. Mehanna and N. D. Sidiropoulos, “Adaptive thresholding for distributed power spectrum sensing,” *Proc. of 38th International Conference on Acoustics, Speech, and Signal Processing*, Vancouver, Canada, May 26-31, 2013