Joint Back-Pressure Power Control and Interference Cancellation in Wireless Multi-Hop Networks

Balasubramanian Gopalakrishnan and Nicholas D. Sidiropoulos, *Fellow, IEEE*

***Abstract*—Back-Pressure Power Control (BPPC) is a cross-layer network optimization policy that uses power control at the physical layer to facilitate efficient routing of packets at the network layer. The starting point of this paper is that interference cancellation can be judiciously employed together with power control to further enhance network throughput. Effective cancellation requires that the interfering signal can be reliably decoded, implying that power control and interference cancellation are tightly coupled. This leads to a joint Back-Pressure Power Control and Interference Cancellation (BPPC-IC) problem formulation, with the pragmatic constraint that each receiver can cancel at most one interfering signal. This problem is shown to be NP-hard, and two approximate solutions are proposed based on successive geometric programming approximation, and an extended weighted minimum mean squared error (WMMSE) reformulation, respectively. A simpler approximation based on stochastic exploration of the control space is also considered. Simulation results demonstrate that joint optimization of power control and interference cancellation pays off, enabling considerably higher network throughput, and lower average delay due to reduced backlogs.**

***Index Terms*—Cross-layer, network optimization, back-pressure, power control, interference cancellation, NP-hard, convex approximation.**

I. Introduction

OVER the past few decades, there have been significant advances in network optimization for maximizing end-to-end throughput [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11]. In parallel to these theoretical developments, there have been remarkable technological advances that have transformed network nodes from simple switches to sophisticated computers, with on-board sensing, control, and communication capabilities. These parallel developments have paved the way towards implementation of advanced network optimization algorithms in the foreseeable future.

Among the various objectives of network optimization, ensuring maximum end-to-end throughput has always been at center stage, reflecting demand for ever higher rates. In a series of papers [1], [2], [3], [4], Tassiulas *et al* established that *back-pressure* network control, a dynamic policy that adapts to changes in differences of adjacent node backlogs, is optimal in terms of achieving maximum stable throughput. That is, back-pressure stabilizes the network, if the network can be stabilized by *any policy* for the given exogenous arrivals. Maximum stable throughput is the maximum aggregate rate of exogenous arrivals that the network can sustain while ensuring that backlogs and delays remain bounded, and therefore payloads are eventually delivered to their destination.

Soon after [1], [2], it was realized that these results are broadly applicable to cross-layer network operations, from scheduling to resource allocation at the physical layer. As an example, back-pressure dictates the right formulation of power control at the physical layer, if one is interested in maximizing throughput at the network layer [12]. This Back-Pressure Power Control (BPPC) problem was revisited in [13], where it was shown that BPPC is NP-hard, and an approximation using a sequential geometric programming strategy was proposed. The approximation in [13] yields substantial gains in terms of throughput over earlier approaches [12].

The insight that motivates this paper is that a strong interferer can be preferable to a weaker one, if it is strong enough to be reliably decoded. That is, if the interfering signal (possibly after de-spreading with its own code) is heard at sufficiently high power relative to other co-channel signals (including the primary signal of interest), then it can be decoded and cancelled. The ability of a receiver to cancel interference depends on the power allocated to the direct and interfering links. Hence interference cancellation and power control are intertwined; joint optimization of transmit powers and interference cancellation 'taps' over all links is needed to exploit the interplay between the two.

Joint power control and interference cancellation for maximum throughput in multi-hop wireless networks has not been considered in the literature, but there is some work addressing related problems at the physical layer. The closest reference is [14], which considered joint power control and interference cancellation to maximize the minimum signal to interference plus noise ratio for a set of co-channel links. The main differences with [14] are in the formulation, including a different cross-layer objective (differential backlog-weighted sum of link rates); and in the solution, where polynomial-time approximation algorithms are proposed, as opposed to [14] which suggests using generic mixed integer linear programming solvers, which have exponential worst-case complexity. As in [14], it is assumed that each link can cancel at most one interferer. The reason for this is that cancelling more than one interferers requires expensive analog to digital converters with very fine resolution and wide dynamic range.

It is shown that BPPC with Interference Cancellation

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The authors are with the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN 55455 USA (e-mail: {gopa0022,nikos}@umn.edu).

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(BPPC-IC) is NP-hard, but it is also shown that judicious approximations yield substantial end-to-end throughput improvements, which can justify the added complexity. Two different approximation strategies are considered: one relying on successive approximation using convex lower-bounding of the link rates [15], [16] and geometric programming; the other using a generalization of the weighted minimum mean squared error (WMMSE) reformulation of the sum-rate minimization problem [17], [18], which renders it in a form suitable for block coordinate descent. We emphasize that the problem considered here is a non-trivial generalization of the weighted sum rate maximization problems considered in [17], [18], [15], [16] (see also [19]), because it incorporates an integer part stemming from the interference cancellation constraints. This makes the problem more difficult.

In addition to full-fledged BPPC-IC, a simpler approximation based on stochastic exploration of the control space is also considered. The motivation for this is as follows. It has been shown in [4] that when the control space is discrete and finite, a greedy randomized policy is optimal from a throughput perspective, at the expense of higher delay. This consists of drawing a random policy per scheduling slot, comparing it to the current one and adopting the new one only if it yields better objective for the present slot. In our present context the control space comprises continuous (powers) and discrete (cancellation taps) variables, and optimization over the continuous variables is NP-hard. While [4] is therefore not applicable in our context, it still motivates considering a random interference cancellation policy (BPPC-RIC) where, in each time slot, each link randomly draws another link as candidate for cancellation. Whether the chosen candidates will be actually cancelled or not is determined by solving a reduced BPPC-IC problem. This solution is compared with the one from the previous slot, and the best one is retained. BPPC-RIC has lower complexity, but takes time to explore the control space.

The merits of BPPC, BPPC-IC and BPPC-RIC are compared in pertinent experiments, demonstrating that joint optimization of power control and interference cancellation pays off, enabling considerably higher network throughput, and lower average delay due to reduced backlogs.

II. System Model

Consider a wireless multi-hop network with N nodes. The network topology is represented by the directed graph $(\mathcal{N}, \mathcal{L})$, where $\mathcal{N} = \{1, \ldots, N\}$ and $\mathcal{L} = \{1, \ldots, L\}$ denote the set of nodes and the set of links respectively. Every link $\ell \in \mathcal{L}$ corresponds to an ordered pair (i, j), where $i \neq j$ and $i, j \in \mathcal{N}$. Let node 1 be the source and node N be the destination - the analysis can be easily extended to multiple sources and multiple destinations, cf. [13]. The aim is to transmit data from the source node to the destination node, possibly using multiple multi-hop routes, in such a way that the throughput from source to destination is maximized.

We assume that the system is slotted in time, with slots indexed by t. Let $W_i(t)$ denote the packet-backlog of node i at the beginning of slot t. The *differential backlog* for link $\ell \leftrightarrow (i, j)$ at the beginning of slot t, $\forall i \in \mathcal{N}\backslash\{N\}$, is defined as [1]

$$D_\ell(t) = \begin{cases} \max\{0, W_i(t) - W_j(t)\}, & j \neq N \\ W_i(t), & j = N. \end{cases} \tag{1}$$

In each time slot, all nodes except the destination node are allowed to transmit data to all nodes other than the source node. Each node may divert its incoming traffic into multiple outgoing links. Let p_ℓ denote the transmission power on link ℓ. Then, the SINR at the receiver of link ℓ is given by

$$\Gamma_\ell = \frac{G_{\ell\ell} p_\ell}{\frac{1}{G_{sg}} \sum_{\substack{k=1 \\ k \neq \ell}}^{L} G_{\ell k} p_k + \sigma_\ell^2}, \forall \ell \in \mathcal{L} \tag{2}$$

where $G_{\ell k}$ denotes the channel gain between the transmitter node of link k and the receiver node of link ℓ, σ_ℓ^2 is the background noise power at the receiver of link ℓ, and G_{sg} represents the spreading / beam-forming / coding gain, if any. The 'overhearing' SINR at the receiver of link ℓ when it *attempts to decode the transmission of link* k, is given by

$$\Gamma_{\ell k} = \frac{G_{\ell k} p_k}{\frac{1}{G_{sg}} \sum_{\substack{m=1 \\ m \neq k}}^{L} G_{\ell m} p_m + \sigma_\ell^2}, \forall k \in \mathcal{L}_{\ell^-} \backslash \{0\}, \forall\, \ell \in \mathcal{L} \tag{3}$$

where $\mathcal{L}_{\ell^-} = \{0\} \bigcup \{\mathcal{L} \backslash \{\ell\}\}$. If $\Gamma_{\ell k}$ is greater than a specified threshold T, then the receiver of link ℓ can reliably decode the transmission of link k and subsequently cancel its contribution from the received signal. We define a set of indicator random variables $\{\{c_{\ell k}\}_{k \in \mathcal{L}_{\ell^-}}\}_{\ell \in \mathcal{L}}$ to denote whether or not link ℓ cancels link k. For $k \neq 0$,

$$c_{\ell k} = \begin{cases} 1, & \text{if link } \ell \text{ cancels link } k \\ 0, & \text{if link } \ell \text{ does not cancel link } k \end{cases} \tag{4}$$

For $k = 0$, $c_{\ell 0} = 1$ corresponds to no interference cancellation by the receiver of link ℓ. So

$$c_{\ell 0} = \begin{cases} 1, & \text{if } c_{\ell k} = 0, \forall k \in \mathcal{L}_{\ell^-} \backslash \{0\} \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

We restrict the receiver of each link $\ell \in \mathcal{L}$ to cancel at most one of its interferers at a time. This translates to a constraint on the cancellation coefficients given by

$$\sum_{\substack{k=0 \\ k \neq \ell}}^{L} c_{\ell k} = 1, \quad \forall\, \ell\, \in\, \mathcal{L} \tag{6}$$

From (4)-(6), it follows that for every $\ell \in \mathcal{L}$, only one of $\{c_{\ell k}\}_{k \in \mathcal{L}_{\ell^-}}$ can be equal to 1. The transmission rate on every link $\ell \in \mathcal{L}$ is upper-bounded by its capacity, given by

$$R_\ell = \sum_{\substack{m=0 \\ m \neq \ell}}^{L} \log\left(1 + c_{\ell m} \gamma_{\ell m}\right) \tag{7}$$

where $c_{\ell m} \in \{0, 1\}$ as per (4)-(5), and $\gamma_{\ell m}$ is the SINR at the receiver of link ℓ after cancelling the transmission on link m when $m \neq 0$, or no cancellation at all when $m = 0$. In both cases, this is given by

$$\gamma_{\ell m} = \frac{G_{\ell\ell} p_\ell}{\frac{1}{G_{sg}} \sum_{\substack{k=1 \\ k \neq \ell, m}}^{L} G_{\ell k} p_k + \sigma_\ell^2}, \forall\, m \in \mathcal{L}_{\ell^-}, \forall\, \ell \in \mathcal{L} \tag{8}$$

III. Problem Formulation

For every t, the link rates are bounded by their respective capacities resulting from the power allocation $\{p_\ell\}_{\ell \in \mathcal{L}}$ and the cancellation coefficients $\{\{c_{\ell k}\}_{k \in \mathcal{L}_{\ell-}}\}_{\ell \in \mathcal{L}}$ obtained at the beginning of that slot. These are determined by solving the optimization problem $\mathbf{\Pi_1}$ that maximizes the differential-backlog weighted sum-rate of the wireless network. This objective function optimizes end-to-end network throughput, as shown in [1], [2], [3], [9].

$\mathbf{\Pi_1}$ BPPC-IC

$$\max_{\substack{\{p_\ell\}_{\ell \in \mathcal{L}} \\ \{\{c_{\ell m}\}_{m \in \mathcal{L}_{\ell-}}\}_{\ell \in \mathcal{L}}}} \sum_{\ell=1}^{L} D_\ell(t) \left[\sum_{\substack{m=0 \\ m \neq \ell}}^{L} \log\left(1 + c_{\ell m}\gamma_{\ell m}\right) \right]$$

$$\text{s.t. } 0 \le p_\ell \le P \quad \forall \ell \in \mathcal{L}, \tag{9a}$$

$$c_{\ell k} \in \{0,1\} \quad \forall k \in \mathcal{L}_{\ell-},\ \forall \ell \in \mathcal{L}, \tag{9b}$$

$$\sum_{\substack{k=0 \\ k \neq \ell}}^{L} c_{\ell k} = 1 \quad \forall \ell \in \mathcal{L} \tag{9c}$$

$$\Gamma_{\ell k} \ge T c_{\ell k} \quad \forall k \in \mathcal{L}_{\ell-} \setminus \{0\},\ \forall \ell \in \mathcal{L} \tag{9d}$$

Constraint (9a) upper bounds the transmission power of each link $\ell \in \mathcal{L}$. Per-node sum power constraints can also be included without changing the nature of the problem. The constraints in (9b)-(9c) reflect the adopted binary cancellation tap model, and restrict the receiver of each link $\ell \in \mathcal{L}$ to cancel *at most* one of its interfering links. The inequalities in (9d) enforce the interference cancellation constraints for $\ell \in \mathcal{L}$. Here T is the SINR threshold for the receiver of a link to reliably decode the data transmitted in another link. For each $k \in \mathcal{L}_{\ell-} \setminus \{0\}$, $\ell \in \mathcal{L}$, the corresponding inequality in (9d) becomes active only when $c_{\ell k} = 1$, and is trivially satisfied when $c_{\ell k} = 0$ (cf. non-negativity of the powers and channel gains). If $c_{\ell k} = 1$, the corresponding constraint in (9d) ensures that the 'overhearing' SINR at the receiver of link ℓ is at least T, when it tries to decode data transmitted in link k and cancel its contribution from the received signal of link ℓ.

The BPPC-IC problem ($\mathbf{\Pi_1}$) is intuitively harder then plain BPPC, which is already NP-hard [13]. This suggests that BPPC-IC is NP-hard too. This is made rigorous in the following.

Claim 1: The BPPC-IC problem $\mathbf{\Pi_1}$ contains all instances of the BPPC problem, and is therefore NP-hard.

Proof: Every instance of the BPPC problem considered in [13] can be characterized by $\{\mathbf{G}, P\}$, where $\mathbf{G}$ is the path-loss matrix $\{G_{\ell k}\}_{\ell, k \in \mathcal{L}}$ and P is the maximum allowable transmission power on each link. On the other hand, the BPPC-IC problem ($\mathbf{\Pi_1}$) can be characterized by $\{\mathbf{G}, P, T\}$, where T is the 'overhearing' SINR threshold. It can be seen from (9a)-(9d) that, for a given $\{\mathbf{G}, P\}$, if the threshold $T > T_{max}(\mathbf{G}, P) = \left[\max_{\substack{k \in \mathcal{L}_{\ell-} \setminus \{0\} \\ \ell \in \mathcal{L}}} \left(\frac{G_{\ell k} P}{\sigma_\ell^2}\right)\right]$, then there is no choice of $\{p_\ell \in [0, P]\}_{\ell \in \mathcal{L}}$ which can support $c_{\ell k} = 1$ for any $k \in \mathcal{L}_{\ell-} \setminus \{0\}$, $\ell \in \mathcal{L}$. Therefore, the optimal cancellation coefficients have to be $\{c^*_{\ell 0} = 1\}_{\ell \in \mathcal{L}}$ and $\{\{c^*_{\ell k} = 0\}_{k \in \mathcal{L}_{\ell-} \setminus \{0\}}\}_{\ell \in \mathcal{L}}$ when $T > T_{max}(\mathbf{G}, P)$. It follows that every problem instance of BPPC, characterized by $\{\mathbf{G}, P\}$, can be posed as a special case of BPPC-IC $\{\mathbf{G}, P, T\}$ with $T > T_{max}(\mathbf{G}, P)$. ■

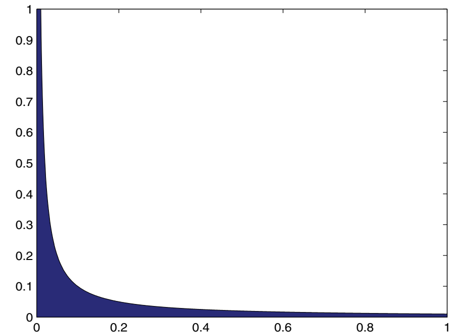

Fig. 1. Illustration of feasible region after interval relaxation for two cancellation coefficients $c_{\ell m}$ vs. $c_{\ell n}$, for $\frac{G_{sg}^2}{T^2} = 10^{-2}$.

A. Interval relaxation

In $\mathbf{\Pi_1}$, the cancellation coefficients $c_{\ell k}$ are restricted to binary values $\{0,1\}$. Rewriting the interference cancellation constraints for link $\ell \in \mathcal{L}$ and candidates $m, n \in \mathcal{L}_{\ell-} \setminus \{0\}$, $m \neq n$ we have,

$$\frac{G_{\ell m} p_m}{\frac{1}{G_{sg}} \sum_{\substack{k=1 \\ k \neq m}}^{L} G_{\ell k} p_k + \sigma_\ell^2} \ge T c_{\ell m} \Rightarrow \frac{G_{\ell m} p_m}{G_{\ell n} p_n} \ge \frac{T c_{\ell m}}{G_{sg}}$$
$$\frac{G_{\ell n} p_n}{\frac{1}{G_{sg}} \sum_{\substack{k=1 \\ k \neq n}}^{L} G_{\ell k} p_k + \sigma_\ell^2} \ge T c_{\ell n} \Rightarrow \frac{G_{\ell n} p_n}{G_{\ell m} p_m} \ge \frac{T c_{\ell n}}{G_{sg}}$$
$$\Rightarrow \frac{T c_{\ell m}}{G_{sg}} \le \frac{G_{\ell m} p_m}{G_{\ell n} p_n} \le \frac{G_{sg}}{T c_{\ell n}} \quad \Rightarrow \quad \boxed{c_{\ell m}\, c_{\ell n} \le \frac{G_{sg}^2}{T^2}} \tag{10}$$

For every $\ell \in \mathcal{L}$, if $c_{\ell m}$ is positive, then the inequality in (10) upper bounds $c_{\ell n}$ to $\frac{G_{sg}^2}{T^2 c_{\ell m}}$ $\forall n \in \mathcal{L}_{\ell-} \setminus \{0, m\}$. In addition, if the SINR threshold T is high, the upper-bound itself is very small, thereby resulting in all cancellation coefficients other than $c_{\ell m}$ to be drawn towards zero. Hence for high T, the cancellation coefficient vector $\{c_{\ell k}\}_{k \in \mathcal{L}_{\ell-} \setminus \{0\}}$ for each link $\ell \in \mathcal{L}$ has at most one significantly positive value and the remaining ones are negligibly small, in comparison. Thus, even if we replace the binary $\{0,1\}$ constraints on the cancellation coefficients with $[0,1]$ interval constraints, the feasible region for these coefficients does not extend to the product space: it will be limited close to the axes. This is illustrated in Fig. 1, and it motivates the relaxation of $\{\{c_{\ell k} \in \{0,1\}\}_{k \in \mathcal{L}_{\ell-}}\}_{\ell \in \mathcal{L}}$ in (9) to $\{\{c_{\ell k} \in [0,1]\}_{k \in \mathcal{L}_{\ell-}}\}_{\ell \in \mathcal{L}}$.

IV. Convex Approximation Algorithms

The modified formulation after interval relaxation is given by $\mathbf{\Pi_2}$ as shown below.

$\mathbf{\Pi_2}$ BPPC-IC

$$\max_{\substack{\{p_\ell\}_{\ell\in\mathcal{L}} \\ \{\{c_{\ell m}\}_{m\in\mathcal{L}_{\ell-}}\}_{\ell\in\mathcal{L}}}} \sum_{\ell=1}^{L} D_\ell(t)\left[\sum_{\substack{m=0\\m\neq\ell}}^{L}\log\left(1+c_{\ell m}\gamma_{\ell m}\right)\right]$$

$$\text{s.t. } 0\le p_\ell\le P \quad \forall \ell\in\mathcal{L}, \tag{11a}$$
$$c_{\ell k}\in[0,1] \quad \forall k\in\mathcal{L}_{\ell-},\ \forall \ell\in\mathcal{L}, \tag{11b}$$
$$\sum_{\substack{k=0\\k\neq\ell}}^{L} c_{\ell k}=1 \quad \forall\ell\in\mathcal{L}, \tag{11c}$$
$$\Gamma_{\ell k}\ge T c_{\ell k} \quad \forall k\in\mathcal{L}_{\ell-}\setminus\{0\},\ \forall\ell\in\mathcal{L} \tag{11d}$$

Using (8), the objective function in $\mathbf{\Pi_2}$ can be rewritten as follows.

$$\sum_{\ell=1}^{L} D_\ell(t)\Bigg[\sum_{\substack{m=0\\m\neq\ell}}^{L}\log\left(G_{\ell\ell}p_\ell c_{\ell m}+\frac{1}{G_{sg}}\sum_{\substack{k=1\\k\neq\ell,m}}^{L}G_{\ell k}p_k+\sigma_\ell^2\right) - \log\left(\frac{1}{G_{sg}}\sum_{\substack{k=1\\k\neq\ell,m}}^{L}G_{\ell k}p_k+\sigma_\ell^2\right)\Bigg]$$

This is a non-convex function of $\{p_\ell\}_{\ell\in\mathcal{L}}$ and $\{\{c_{\ell m}\}_{m\in\mathcal{L}_{\ell-}}\}_{\ell\in\mathcal{L}}$. In this paper, we present two methods to find the approximate link-powers $\{p_\ell\}_{\ell\in\mathcal{L}}$ and cancellation coefficients $\{\{c_{\ell k}\}_{k\in\mathcal{L}_{\ell-}}\}_{\ell\in\mathcal{L}}$ for $\mathbf{\Pi_2}$. In the first method, we approximate the non-concave objective function by a concave lower bound, followed by a variable transformation, which yields a concave maximization problem. In the second approach, we extend the weighted minimum mean squared error (WMMSE) framework for weighted sum-rate maximization problem) [17],[18] to our formulation in $\mathbf{\Pi_2}$, and use block coordinate descent to obtain approximate $\{p_\ell\}_{\ell\in\mathcal{L}}$ and $\{\{c_{\ell k}\}_{k\in\mathcal{L}_{\ell-}}\}_{\ell\in\mathcal{L}}$.

A. Successive Approximation algorithm

In this algorithm, we approximate each term inside the inner summation of the objective function in $\mathbf{\Pi_2}$ by the following concave lower bound, first introduced in [15], [16].

$$\log(1+z)\ge\alpha\log(z)+\beta \text{ for } \begin{cases}\alpha=\frac{z_o}{1+z_o},\\ \beta=\log(1+z_o)-\frac{z_o}{1+z_o}\log(z_o)\end{cases} \tag{12}$$

This bound is tight at $z=z_o$ and as $z_o\to\infty$, the inequality becomes $\log(1+z)\ge\log(z)$. Introducing a logarithmic transformation of variables, $\tilde{p}_\ell=\log p_\ell$, $\tilde{c}_{\ell k}=\log c_{\ell k}$ and applying (12) to $\mathbf{\Pi_2}$, we get the following approximate formulation[1] $\mathbf{\Pi_3}$.

[1]Even though we are maximizing a lower bound to the original objective function in Π_1, the solution to Π_3 yields an approximate solution (which may give an objective value higher than the maximum of the original objective in Π_1), as a result of the interval relaxation introduced for $\{\{c_{\ell k}\}_{k\in\mathcal{L}_{\ell-}}\}_{\ell\in\mathcal{L}}$ in Π_2

$\mathbf{\Pi_3}$

$$\max_{\substack{\{\tilde{p}_\ell\}_{\ell\in\mathcal{L}} \\ \{\{\tilde{c}_{\ell m}\}_{m\in\mathcal{L}_{\ell-}}\}_{\ell\in\mathcal{L}}}} \sum_{\ell=1}^{L} D_\ell(t)\Bigg[\sum_{\substack{m=0\\m\neq\ell}}^{L}\alpha_{\ell m}(t)\Big(\tilde{G}_{\ell\ell}+\tilde{G}_{sg}+\tilde{p}_\ell - \log\Big[\sum_{\substack{k=1\\k\neq\ell,m}}^{L}e^{\tilde{G}_{\ell k}+\tilde{p}_k}+e^{\tilde{\sigma}_\ell^2}\Big]+\tilde{c}_{\ell m}\Big)+\beta_{\ell m}(t)\Bigg]$$

$$\text{s.t. } \tilde{p}_\ell\le\tilde{P}=\log(P) \quad \forall\ell\in\mathcal{L}, \tag{13a}$$
$$\tilde{c}_{\ell k}\in[\log(\varepsilon),0] \quad \forall k\in\mathcal{L}_{\ell-},\ \forall\ell\in\mathcal{L}, \tag{13b}$$
$$\log\left(\sum_{\substack{k=0\\k\neq\ell}}^{L}e^{\tilde{c}_{\ell k}}\right)\le\varepsilon \quad \forall\ell\in\mathcal{L}, \tag{13c}$$
$$\tilde{G}_{\ell k}+\tilde{G}_{sg}+\tilde{p}_k-\log\left(\sum_{\substack{j=1\\j\neq k}}^{L}e^{\tilde{G}_{\ell j}+\tilde{p}_j}+e^{\tilde{\sigma}_\ell^2}\right) \ge \tilde{T}+\tilde{c}_{\ell k} \quad \forall k\in\mathcal{L}_{\ell-},\forall\ell\in\mathcal{L}. \tag{13d}$$

where $\alpha_{\ell m}(t),\beta_{\ell m}(t)$ are constants chosen to bound $\log(1+c_{\ell m}\gamma_{\ell m})$ according to (12) at a suitable supporting point determined as discussed in section IV-A, $\tilde{G}_{\ell k}:=\log\left(\frac{G_{\ell k}}{G_{sg}}\right)$, $\tilde{\sigma}_\ell^2:=\log(\sigma_\ell^2)$ and $\varepsilon>0$ is a small positive constant introduced to avoid numerical errors (since $\log(0)=-\infty$).

The objective function $\mathbf{\Pi_3}$ is concave, since it comprises a sum of linear and concave functions of $\{\tilde{p}_\ell\}_{\ell\in\mathcal{L}}$ and $\{\{\tilde{c}_{\ell m}\}_{m\in\mathcal{L}_{\ell-}}\}_{\ell\in\mathcal{L}}$. The weights $D_\ell(t)$ and constants $\alpha_{\ell m}(t)$, $\beta_{\ell m}(t)$ are nonnegative, cf. (1), (12). Furthermore, the set of constraints for $\mathbf{\Pi_3}$ are convex with respect to $\{\tilde{p}_\ell\}_{\ell\in\mathcal{L}}$ and $\{\{\tilde{c}_{\ell k}\}_{k\in\mathcal{L}_{\ell-}}\}_{\ell\in\mathcal{L}}$. Thus, $\mathbf{\Pi_3}$ involves the maximization of a concave function over a convex feasible set which is a convex optimization problem.

1) Algorithm Description: For every time slot t, problem $\mathbf{\Pi_3}$ is solved using a variation of the Batch Successive Approximation algorithm in [13]. For the first iteration, $\mathbf{\Pi_3}$ is solved with $\alpha_{\ell m}(t)=1$ and $\beta_{\ell m}(t)=0, \forall m\in\mathcal{L}_{\ell-}, \forall\ell\in\mathcal{L}$. Then, for subsequent iterations, using (12), (14), the values of $\alpha_{\ell m}(t)$ and $\beta_{\ell m}(t)$, $\forall m\in\mathcal{L}_{\ell-}\ \forall\ell\in\mathcal{L}$ are updated using $\{\tilde{p}_\ell^*\}_{\ell\in\mathcal{L}}$, $\{\{\tilde{c}_{\ell k}^*\}_{k\in\mathcal{L}_{\ell-}}\}_{\ell\in\mathcal{L}}$ obtained from the previous iteration. This tightens the individual link rate lower bounds so that they coincide with the link rates at $\{\tilde{p}_\ell^*\}_{\ell\in\mathcal{L}}$ and $\{\{\tilde{c}_{\ell k}^*\}_{k\in\mathcal{L}_{\ell-}}\}_{\ell\in\mathcal{L}}$. This procedure is repeated until the objective function value converges. The description of the algorithm is given below.

Algorithm 1: Successive Approximation for BPPC-IC

1) *Initialization step:* For each time slot t, calculate $D_\ell(t)$, reset iteration counter $n=1$, and set $\alpha_{\ell m}^{(n)}(t)=1$ and $\beta_{\ell m}^{(n)}(t)=0, \forall m\in\mathcal{L}_{\ell-}, \forall\ell\in\mathcal{L}$.
2) repeat
3) *Optimization Step:* Solve $\mathbf{\Pi_3}\to\{\tilde{p}^{*^{(n)}}\}_{\ell\in\mathcal{L}}$, $\{\{\tilde{c}_{\ell k}^{*^{(n)}}\}_{k\in\mathcal{L}_{\ell-}}\}_{\ell\in\mathcal{L}}$.
4) *Update step:* Update $\alpha_{\ell m}^{(n+1)}(t)$, $\beta_{\ell m}^{(n+1)}(t)$ according to (12) for each $m\in\mathcal{L}_{\ell-}$ and $\forall\ell\in\mathcal{L}$ at the supporting

point

$$z_0 = \left(\frac{G_{\ell\ell}\, p_\ell^{*(n)} c_{\ell m}^{*(n)}}{\frac{1}{G_{sg}} \sum_{\substack{k=1 \\ k \neq \ell, m}}^{L} G_{\ell k}\, p_k^{*(n)} + \sigma_\ell^2} \right) \tag{14}$$

where $p_\ell^{*(n)} = e^{\tilde{p}_\ell^{*(n)}}$ and $c_{\ell m}^{*(n)} = e^{\tilde{c}_{\ell m}^{*(n)}}$, $\forall m \in \mathcal{L}_{\ell-}, \forall \ell \in \mathcal{L}$

5) $n = n + 1$
6) until convergence of the objective value (within ε-accuracy)
7) *Rounding step:* $c_{\ell k}^* = 1$, $c_{\ell m}^* = 0, \forall m \neq k, m \in \mathcal{L}_{\ell-}$ where $k = \arg\max_{m \in \mathcal{L}_{\ell-}} \left(\frac{G_{\ell\ell}\, p_\ell^* c_{\ell m}^*}{\frac{1}{G_{sg}} \sum_{\substack{j=1 \\ j \neq \ell, m}}^{L} G_{\ell j}\, p_j^* + \sigma_\ell^2} \right)$

Here $\alpha_{\ell m}^{(n)}(t), \beta_{\ell m}^{(n)}(t), p_\ell^{*(n)}, c_{\ell m}^{*(n)}$ represents the values of the variables in the n^{th} iteration. The optimization variables for BPPC-IC are L link powers and $L(L-1)$ cancellation coefficients. Hence the worst-case complexity [13] is $\mathcal{O}(L^7)$, where $L = |\mathcal{L}|$.

2) Random Interference Cancellation Policy: When the control space is discrete and finite, it was shown in [4] that a simple randomized policy is - surprisingly - optimal from a throughput point of view (the price paid is excess delay). Although the result of [4] does not apply directly to our problem (involving both continuous and discrete optimization variables), it motivates a comparison of the BPPC-IC policy to what we will refer to as BPPC-RIC (for Randomized Interference Cancellation). Here, we present two randomized policies namely a) *BPPC-RIC1* and b) *BPPC-RIC2*.

BPPC-RIC1: For t = 1, we fix $\{c_{\ell 0}(t) = 1\}_{\ell \in \mathcal{L}}$ and $\{\{c_{\ell k}(t) = 0\}_{k \in \mathcal{L}_{\ell-} \setminus \{0\}}\}_{\ell \in \mathcal{L}}$ and obtain the values of $\{p_\ell(t)\}_{\ell \in \mathcal{L}}$ by solving the BPPC problem in [13]. For t > 1, the receiver of every link $\ell \in \mathcal{L}$ chooses a link at random, say $k_\ell \in \mathcal{L}_{\ell-} \setminus \{0\}$, for cancellation ($c_{\ell k_\ell}(t) = 1$) and fixes the remaining coefficients $\{c_{\ell m}(t)\}_{m \in \mathcal{L}_{\ell-} \setminus \{k_\ell\}}$ to zero. If $\mathbf{\Pi_4}$ is infeasible for this choice of $\{\{c_{\ell m}(t)\}_{m \in \mathcal{L}_{\ell-}}\}_{\ell \in \mathcal{L}}$, we simply reuse the power vector and cancellation taps from the previous slot in the current slot as well. On the other hand, if $\mathbf{\Pi_4}$ is feasible, it is solved to obtain $\{p_\ell^*(t)\}_{\ell \in \mathcal{L}}$. Now, the objective function for the current time slot is evaluated using $\left(\{p_\ell^*(t)\}_{\ell \in \mathcal{L}}, \{\{c_{\ell m}(t)\}_{m \in \mathcal{L}_{\ell-}}\}_{\ell \in \mathcal{L}}\right)$ and $\left(\{p_\ell^{opt}(t-1)\}_{\ell \in \mathcal{L}}, \{\{c_{\ell m}^{opt}(t-1)\}_{m \in \mathcal{L}_{\ell-}}\}_{\ell \in \mathcal{L}}\right)$, the pair selected for slot $t-1$. The pair achieving the higher objective value is selected for the present time slot.

Π_4 BPPC-RIC1

$$\max_{\{\tilde{p}_\ell\}_{\ell \in \mathcal{L}}} \sum_{\ell=1}^{L} D_\ell(t) \Bigg[\alpha_{\ell k_\ell}(t) \Big(\tilde{G}_{\ell\ell} + \tilde{G}_{sg} + \tilde{p}_\ell - \log \Big[\sum_{\substack{j=1 \\ j \neq \ell, k_\ell}}^{L} e^{\tilde{G}_{\ell j} + \tilde{p}_j} + e^{\tilde{\sigma}_\ell^2} \Big] \Big) + \beta_{\ell k_\ell}(t) \Bigg]$$

$$\text{s.t.} \quad \tilde{p}_\ell \leq \tilde{P} := \log P \quad \forall \ell \in \mathcal{L}, \tag{15a}$$

$$\tilde{G}_{\ell k_\ell} + \tilde{G}_{sg} + \tilde{p}_{k_\ell} - \log \Big[\sum_{\substack{k=1 \\ k \neq k_\ell}}^{L} e^{\tilde{G}_{\ell k_\ell} + \tilde{p}_{k_\ell}} + e^{\tilde{\sigma}_\ell^2} \Big] \geq \tilde{T}, \ \forall \ell \in \mathcal{L}. \tag{15b}$$

For such randomized exploration of the cancellation part of the control space, the probability of $\mathbf{\Pi_4}$ being infeasible grows rapidly with the number of links. As a result, the performance of BPPC-RIC1 tends towards that of BPPC. One could think of pruning the set of candidates for cancellation down to nearest neighbors, but note that the stronger interferers are determined not only by proximity but also by transmission powers, which are optimization variables that are not known *a priori*. In order to boost the benefit of BPPC-RIC over BPPC, we propose an improved version that simultaneously explores $2^{|L|}$ cancellation configurations.

BPPC-RIC2: At the start of every time slot, the receiver of each link $\ell \in \mathcal{L}$ chooses one other link at random, say $k_\ell \in \mathcal{L}_{\ell-}$ as a *candidate* for cancellation, i.e., $\{c_{\ell k}\}_{k \in \{0, k_\ell\}} \in [\varepsilon, 1]$ (ε for avoiding numerical errors), and fixes the remaining coefficients $\{c_{\ell k}\}_{k \in \mathcal{L}_{\ell-} \setminus \{0, k_\ell\}}$ to zero. Here each receiver retains the freedom to choose between cancelling the randomly chosen link, or not, unlike BPPC-RIC1 where all cancellation taps are fixed. Then $\mathbf{\Pi_3}$ is solved with the aforementioned constraints on $c_{\ell k}$, to obtain the optimal values of $\{\tilde{p}_\ell^*(t)\}_{\ell \in \mathcal{L}}$ and $\{\{\tilde{c}_{\ell m}^*(t)\}_{m \in \{0, k_\ell\}}\}_{\ell \in \mathcal{L}}$. The value of the objective function for the present time slot attained by $\{\tilde{p}_\ell^*(t)\}_{\ell \in \mathcal{L}}$ and $\{\{\tilde{c}_{\ell m}^*(t)\}_{m \in \mathcal{L}_{\ell-}}\}_{\ell \in \mathcal{L}}$ is compared to that achieved by $\{\tilde{p}_\ell^{opt}(t-1)\}_{\ell \in \mathcal{L}}$ and $\{\{\tilde{c}_{\ell m}^{opt}(t-1)\}_{m \in \mathcal{L}_{\ell-}}\}_{\ell \in \mathcal{L}}$ (power allocation and cancellation coefficients for slot $t-1$) and the pair resulting in the higher value is chosen for the present time slot. The BPPC-RIC2 policy effectively explores 2^L cancellation configurations per slot, thus being in-between of BPPC-RIC1 and BPPC-IC in terms of complexity. The optimization variables for BPPC-RIC2 are L link powers and $2L$ cancellation coefficients. Hence the worst-case complexity is $\mathcal{O}(L^{3.5})$ [13].

3) Extension to multiple flows: In multi-flow scenarios, each node maintains separate queues for different destinations (flows) in the network (note that different sources sending to one destination can be treated as a single flow). In each time slot, each link is fully committed to the flow with the maximum differential backlog over that link, i.e., back pressure routing is a winner-takes-all policy [1] for each link, and the winner's maximum differential backlog is what is used as the link weight in the BPPC-IC formulation. Hence our algorithms can be applied verbatim in the (so-called *multi-*

commodity) multiple-flows case, the only difference is that the determination of link weights prior to optimization is different. See also [13].

B. Extended WMMSE algorithm

An iterative algorithm for weighted sum-rate maximization which works by minimizing a weighted mean square error (MSE) function instead, has been developed by Shi *et al.* in [18], extending earlier work in [17]. The merit of this technique lies in that the stationary point of the reformulated problem can be approached using a series of *closed form* updates, which significantly reduces complexity relative to the successive convex approximation algorithm. Here, we extend this technique to find an approximate solution of the BPPC-IC problem in $\mathbf{\Pi_2}$ by iteratively solving an equivalent weighted MMSE problem as shown below.

Define $v_\ell = \sqrt{p_\ell},\ \forall\ \ell \in \mathcal{L}$ and $H_{\ell k} = \sqrt{G_{\ell k}}\ \forall\ k, \ell \in \mathcal{L}$. Now, consider the following weighted MSE objective function.

$$f_{wmmse} = \sum_{\ell \in \mathcal{L}} D_\ell(t) \sum_{m \in \mathcal{L}_{\ell-}} \left(w_{\ell m}\ e_{\ell m}(u_{\ell m}, c_{\ell m}, \mathbf{v}) - \log w_{\ell m}\right) \tag{16}$$

where $D_\ell(t)$ is the differential backlog for link ℓ at time slot t, $\mathbf{v} = [v_1, v_2, \ldots, v_L]^T$ and the error function $e_{\ell m}$ is defined as

$$\begin{aligned} e_{\ell m}(u_{\ell m}, c_{\ell m}, \mathbf{v}) = (u_{\ell m} H_{\ell\ell}\sqrt{c_{\ell m}} v_\ell - 1)^2 + \\ \frac{1}{G_{sg}} \sum_{\substack{k=1 \\ k \neq l,m}}^{L} (u_{\ell m} H_{\ell k} v_k)^2 + \sigma_\ell^2 u_{\ell m}^2,\ \forall m \in \mathcal{L}_{\ell-}\ \forall \ell \in \mathcal{L} \end{aligned} \tag{17}$$

In (16) and (17) $w_{\ell m} \in \mathcal{R}^+$ and $u_{\ell m} \in \mathcal{R}$ are auxiliary variables which are introduced in order to facilitate the splitting of the main problem into simpler sub-problems. Here $e_{\ell m}(u_{\ell m}, c_{\ell m}, \mathbf{v})$ can be considered as the MSE between a transmitted signal at link $\ell \in \mathcal{L}$, of unit power, and the received signal at the Rx of link ℓ after cancelling the interference from link $m \in \mathcal{L}_{\ell-} \backslash \{0\}$. The scalars v_ℓ and $u_{\ell m}$ assume the role of transmit precoder and equalizer weights, respectively, and σ_ℓ^2 is the background noise power. Consider

$\mathbf{\Pi_5}$

$$\min_{\mathbf{u},\mathbf{v},\mathbf{w},\mathbf{c}} \sum_{\ell \in \mathcal{L}} D_\ell(t) \sum_{m \in \mathcal{L}_{\ell-}} \Big(w_{\ell m}\ e_{\ell m}(u_{\ell m}, c_{\ell m}, \mathbf{v}) - \log w_{\ell m}\Big)$$

$$\text{s.t.}\quad 0 \leq v_\ell^2 \leq P \quad \forall \ell \in \mathcal{L}, \tag{18a}$$

$$c_{\ell k} \in [0,1] \quad \forall k \in \mathcal{L}_{\ell-},\ \forall \ell \in \mathcal{L}, \tag{18b}$$

$$\sum_{k \in \mathcal{L}_{\ell-}} c_{\ell k} = 1 \quad \forall \ell \in \mathcal{L}, \tag{18c}$$

$$\frac{H_{\ell k}^2 v_k^2}{\frac{1}{G_{sg}} \sum_{\substack{m=1 \\ m \neq k}}^{L} H_{\ell m}^2 v_m^2 + \sigma_\ell^2} \geq T c_{\ell k} \tag{18d}$$

$$\forall k \in \mathcal{L}_{\ell-} \setminus \{0\},\ \forall \ell \in \mathcal{L}.$$

where $\mathbf{u} = \{\{u_{\ell m}\}_{m \in \mathcal{L}_{\ell-}}\}_{\ell \in \mathcal{L}}$, $\mathbf{w} = \{\{w_{\ell m}\}_{m \in \mathcal{L}_{\ell-}}\}_{\ell \in \mathcal{L}}$ and $\mathbf{c} = \{\{c_{\ell m}\}_{m \in \mathcal{L}_{\ell-}}\}_{\ell \in \mathcal{L}}$. Since $\mathbf{w}$ and $\mathbf{u}$ do not appear in the constraints, their optimal values can be obtained by setting the partial derivatives of the objective function with respect to $w_{\ell m}$ and $u_{\ell m}$ to zero. Therefore $\forall m \in \mathcal{L}_{\ell-}$, $\forall \ell \in \mathcal{L}$,

$$\begin{aligned} u_{\ell m}^* &= \frac{H_{\ell\ell}\sqrt{c_{\ell m}} v_\ell}{\left(H_{\ell\ell}\sqrt{c_{\ell m}} v_\ell\right)^2 + \frac{1}{G_{sg}} \sum_{\substack{k=1 \\ k \neq \ell,m}}^{L} (H_{\ell k} v_k)^2 + \sigma_\ell^2}, \\ w_{\ell m}^* &= \left(1 - u_{\ell m}^* H_{\ell\ell}\sqrt{c_{\ell m}} v_\ell\right)^{-1} \end{aligned} \tag{19}$$

Substituting (19) into (17), we get

$$\begin{aligned} e_{\ell m}(u_{\ell m}^*, c_{\ell m}, \mathbf{v}) &= \left(1 + \frac{H_{\ell\ell}^2 v_\ell^2 c_{\ell m}}{\frac{1}{G_{sg}} \sum_{\substack{k=1 \\ k \neq l,m}}^{L} H_{\ell k}^2 v_k^2 + \sigma_\ell^2}\right)^{-1} \\ &= (1 - u_{\ell m}^* H_{\ell\ell}\sqrt{c_{\ell m}} v_\ell) \end{aligned} \tag{20}$$

Substituting (20) and (19) into (18), we get an optimization problem in terms of $\mathbf{v}$ and $\mathbf{c}$ as shown below

$\mathbf{\Pi_6}$

$$\max_{\mathbf{v},\mathbf{c}} \sum_{\ell \in \mathcal{L}} D_\ell(t) \sum_{m \in \mathcal{L}_{\ell-}} \log\left(1 + \frac{H_{\ell\ell}^2 v_\ell^2 c_{\ell m}}{\frac{1}{G_{sg}} \sum_{\substack{k=1 \\ k \neq l,m}}^{L} H_{\ell k}^2 v_k^2 + \sigma_\ell^2}\right)$$

$$\text{s.t.}\quad 0 \leq v_\ell^2 \leq P \quad \forall \ell \in \mathcal{L}, \tag{21a}$$

$$c_{\ell k} \in [0,1] \quad \forall k \in \mathcal{L}_{\ell-},\ \forall \ell \in \mathcal{L}, \tag{21b}$$

$$\sum_{k \in \mathcal{L}_{\ell-}} c_{\ell k} = 1 \quad \forall \ell \in \mathcal{L}, \tag{21c}$$

$$\frac{H_{\ell k}^2 v_k^2}{\frac{1}{G_{sg}} \sum_{\substack{m=1 \\ m \neq k}}^{L} H_{\ell m}^2 v_m^2 + \sigma_\ell^2} \geq T c_{\ell k} \tag{21d}$$

$$\forall k \in \mathcal{L}_{\ell-} \setminus \{0\}, \forall \ell \in \mathcal{L}.$$

This optimization problem is the same as $\mathbf{\Pi_2}$. Hence the weighted MSE minimization problem in $\mathbf{\Pi_5}$ is equivalent to the differential backlog weighted sum-rate maximization in $\mathbf{\Pi_2}$. Furthermore, the equivalency of stationary points can be shown in terms of the KKT conditions, see Appendix I.

1) Block coordinate descent approach for weighted MSE minimization: Block coordinate descent can now be used to obtain approximate $\{p_\ell^*\}_{\ell \in \mathcal{L}}$ and $\{\{c_{\ell m}^*\}_{m \in \mathcal{L}_{\ell-}}\}_{\ell \in \mathcal{L}}$ for $\mathbf{\Pi_6}$. In each step, we update a set of the variables keeping the rest fixed. This updating procedure is repeated until convergence of the cost function. The update steps are as outlined below.

For every t, we initialize $v_\ell = \sqrt{P}$, $\forall \ell \in \mathcal{L}$ and $c_{\ell 0} = 1$, $c_{\ell k} = 0$, $\forall k \in \mathcal{L}_{\ell-} \backslash \{0\}$, $\forall \ell \in \mathcal{L}$

u-update:

$$\frac{\partial f_{wmmse}}{\partial u_{\ell m}} = 0 \Rightarrow$$

$$u_{\ell m}^* = \frac{H_{\ell\ell}\sqrt{c_{\ell m}} v_\ell}{\left(H_{\ell\ell}\sqrt{c_{\ell m}} v_\ell\right)^2 + \frac{1}{G_{sg}} \sum_{\substack{k=1 \\ k \neq \ell,m}}^{L} (H_{\ell k} v_k)^2 + \sigma_\ell^2},$$

$$\forall m \in \mathcal{L}_{\ell-}, \forall \ell \in \mathcal{L} \tag{22}$$

w-update:

$$\frac{\partial f_{wmmse}}{\partial w_{\ell m}} = 0 \Rightarrow w^*_{\ell m} = (1 - u^*_{\ell m} H_{\ell\ell}\sqrt{c_{\ell m}} v_\ell)^{-1} \quad (23)$$
$$\forall m \in \mathcal{L}_{\ell-},\ \forall \ell \in \mathcal{L}$$

v-update: Here, we fix $u_{\ell m}, w_{\ell m}$ and $c_{\ell m}, \forall m \in \mathcal{L}_{\ell-}, \forall \ell \in \mathcal{L}$. Then the update of $\mathbf{v}$ can be formulated as shown in $\mathbf{\Pi_7}$. This is not a convex formulation, since the interference cancellation constraints are non-convex in $\mathbf{v}$. But we use the non-negativity constraint explained below to convert $\mathbf{\Pi_7}$ into a convex problem.

$$\mathbf{\Pi_7} \qquad \min_{\mathbf{v}} \sum_{\ell=1}^{L} D_\ell \sum_{m \in \mathcal{L}_{\ell-}} \left(w^*_{\ell m} e_{\ell m}(u^*_{\ell m}, c_{\ell m}, \mathbf{v}) \right)$$

$$\text{s.t.} \quad v_\ell^2 \le P \quad \forall \ell \in \mathcal{L}, \quad (24a)$$
$$\frac{G_{\ell k} v_k^2}{\frac{1}{G_{sg}} \sum_{j \neq k,\, j=1}^{L} G_{\ell j} v_j^2 + \sigma_\ell^2} \ge T c_{\ell k} \forall k \in \mathcal{L}_{\ell-},\ \forall \ell \in \mathcal{L}. \quad (24b)$$

Non-negativity constraint on $\{v_\ell\}_{\ell\in\mathcal{L}}$: It can be seen from the update of u's in (22), that the $u^*_{\ell m}$ and v_ℓ have the same sign. Hence, on substituting $u^*_{\ell m}$ into (17), problem $\mathbf{\Pi_7}$ becomes a function of $\{v_\ell^2\}_{\ell\in\mathcal{L}}$. It follows that, without loss of generality $\{v_\ell\}_{\ell\in\mathcal{L}}$ can be assumed to be non-negative. Thus $\mathbf{\Pi_7}$ can be reformulated as a convex optimization problem $\mathbf{\Pi_8}$ with a quadratic objective and a second-order cone constraint, which is solved to obtain $\{v^*_\ell\}_{\ell\in\mathcal{L}}$.

$$\mathbf{\Pi_8} \qquad \min_{\mathbf{v}} \sum_{\ell=1}^{L} D_l \sum_{m \in \mathcal{L}_{\ell-}} w_{\ell m} e_{\ell m}(u^*_{\ell m}, c_{\ell m}, \mathbf{v})$$

$$\text{s.t.} \quad v_\ell^2 \le P \quad \forall \ell \in \mathcal{L}, \quad (25a)$$
$$v_\ell \ge 0 \quad \forall \ell \in \mathcal{L}, \quad (25b)$$
$$\|\mathbf{v_t^{\ell k}}\| \le \left(\sqrt{G_{\ell k} + \frac{T G_{\ell k} c_{\ell k}}{G_{sg}}} \right) v_k$$
$$\forall k \in \mathcal{L}_{\ell-} \backslash \{0\}\ \forall \ell \in \mathcal{L}. \quad (25c)$$

where $\mathbf{v_t^{\ell k}} = \left[\{ \sqrt{\frac{G_{\ell m} T c_{\ell k}}{G_{sg}}} v_m \}_{m\in\mathcal{L}},\ \sqrt{\sigma_\ell^2 T c_{\ell k}} \right]^T$

c-update: From $\{v^*_\ell\}_{\ell\in\mathcal{L}}$ obtained from $\mathbf{\Pi_8}$, we get the values of $\{p^*_\ell\}_{\ell\in\mathcal{L}}$ using $p^*_\ell = (v^*_\ell)^2$. We now move back to the optimization in $\mathbf{\Pi_2}$ or equivalently $\mathbf{\Pi_6}$ and update $\mathbf{c}$ using $\mathbf{\Pi_9}$, after fixing $p_\ell = p^*_\ell,\ \forall \ell \in \mathcal{L}$. The optimization for the c-update step is equivalent to a water-filling problem under a spectral mask constraint, with $\{\{c_{\ell m}\}_{m\in\mathcal{L}_{\ell-}}\}_{\ell\in\mathcal{L}}$ assuming the role of the powers. With $\{p_\ell\}_{\ell\in\mathcal{L}}$ fixed, the objective function is concave in $\{\{c_{\ell m}\}_{m\in\mathcal{L}_{\ell-}}\}_{\ell\in\mathcal{L}}$, and the constraints are linear in $\{\{c_{\ell m}\}_{m\in\mathcal{L}_{\ell-}}\}_{\ell\in\mathcal{L}}$. The solution to $\mathbf{\Pi_9}$ can be obtained [20] by dualizing only the sum constraint (28b) in a manner similar to classical water-filling, and the end result is given by

$$c^*_{\ell 0} = \left[\frac{1}{\mu^*_\ell} - \frac{1}{\gamma_{\ell 0}} \right]_0^1, \forall \ell \in \mathcal{L} \quad (26)$$
$$c^*_{\ell m} = \left[\frac{1}{\mu^*_\ell} - \frac{1}{\gamma_{\ell m}} \right]_0^{\min\left(\frac{\Gamma_{\ell m}}{T}, 1\right)}, \forall m \in \mathcal{L}_{\ell-},\ \forall \ell \in \mathcal{L} \quad (27)$$

where $\Gamma_{\ell m}$ and $\gamma_{\ell m}$ are defined in (8) and (3) respectively, and μ^*_ℓ can be found using bisection to ensure that $\sum_{m\in\mathcal{L}_{\ell-}} c^*_{\ell m} = \min\left(1, \sum_{m\in\mathcal{L}_{\ell-}\backslash\{0\}} \min\left(\frac{\Gamma_{\ell m}}{T}, 1\right)\right)$.

Condition for near-binary cancellation coefficients: In order to simplify the analysis of BPPC-IC problem $\mathbf{\Pi_1}$, the binary constraints on the cancellation coefficients were replaced with interval constraints. However it can be shown that for $\frac{D_\ell}{g_\lambda(\varepsilon)}\left(\frac{\gamma_{\ell m}}{1+\gamma_{\ell m}\varepsilon} - \frac{\gamma_{\ell 0}}{1+(1-\varepsilon)\gamma_{\ell 0}}\right) \le T < \Gamma_\ell^{\max}$, where $\Gamma_\ell^{\max} = \max_{\substack{k\in\mathcal{L}_{\ell-}\backslash\{0\} \\ T<\Gamma_{\ell k}}} \Gamma_{\ell k}$, $\varepsilon \in [0, \varepsilon_{up}]$, $\varepsilon_{up} = \left[\left(\sqrt{\left(1+\frac{1}{\gamma_{\ell 0}}\right)\frac{1}{\gamma_{\ell m}}} \right) \frac{\sqrt{1+\frac{1}{\gamma_{\ell 0}}} - \sqrt{\frac{1}{\gamma_{\ell m}}}}{\sqrt{1+\frac{1}{\gamma_{\ell 0}}} + \sqrt{\frac{1}{\gamma_{\ell m}}}} \right]$ and $g_\lambda(\varepsilon) = \frac{D_\ell}{\Gamma_{\ell m}}\left[\frac{\varepsilon}{\varepsilon+\frac{1}{\gamma_{\ell m}}} + \frac{\varepsilon}{\varepsilon - \frac{1+\gamma_{\ell 0}}{\gamma_{\ell 0}}} \right]$, it is possible to obtain near-binary cancellation coefficients as solution to $\mathbf{\Pi_9}$ (proof - see Appendix II).

$$\mathbf{\Pi_9}$$
$$\max_{\mathbf{c}} \sum_{\ell=1}^{L} D_\ell \sum_{m \in \mathcal{L}_{\ell-}} \log\left(1 + \frac{G_{\ell\ell} p_\ell c_{\ell m}}{\frac{1}{G_{sg}} \sum_{k\neq \ell, m} G_{\ell k} p_k + \sigma_\ell^2} \right)$$

$$\text{s.t.} \quad c_{\ell m} \in [0,1] \quad \forall m \in \mathcal{L}_{\ell-} \quad \forall \ell \in \mathcal{L}, \quad (28a)$$
$$\sum_{m \in \mathcal{L}_{\ell-}} c_{\ell m} \le 1 \quad \forall \ell \in \mathcal{L}, \quad (28b)$$
$$c_{\ell k} \le \frac{1}{T} \frac{G_{\ell k} p_k}{\frac{1}{G_{sg}} \sum_{j\neq k,\, j=1}^{L} G_{\ell j} p_j + \sigma_\ell^2}, \forall k \in \mathcal{L}_{\ell-}, \forall \ell \in \mathcal{L}. \quad (28c)$$

Algorithm 2: Extended WMMSE algorithm to solve BPPC-IC

1) *Initialization* - For each time slot t, calculate the differential backlogs, reset iteration counter $n = 1$, and set $v_\ell^{(n)} = \sqrt{P}$, $\forall \ell \in \mathcal{L}$, $c_{\ell 0}^{(n)} = 1$, $c_{\ell m}^{(n)} = 0\ \forall m \in \mathcal{L}_{\ell-}, \forall \ell \in \mathcal{L}$.
2) repeat
3) *u-update* - using (22)
4) *w-update* - using (23)
5) *v-update* - Solve $\mathbf{\Pi_8}$ to obtain the updated $\{p_\ell\}_{\ell\in\mathcal{L}}$.
6) *c-update* - Solve $\mathbf{\Pi_9}$ to obtain the updated $\{\{c_{\ell m}\}_{m\in\mathcal{L}_{\ell-}}\}_{\ell\in\mathcal{L}}$.
7) n = n+1
8) until $|\log(w_{\ell m}^{(n)}) - \log(w_{\ell m}^{(n-1)})| \le \epsilon, \forall m \in \mathcal{L}_{\ell-}, \forall \ell \in \mathcal{L}$.

In the WMMSE framework, we increase the problem dimension by introducing auxiliary variables $\mathbf{u}$ and $\mathbf{w}$ to break

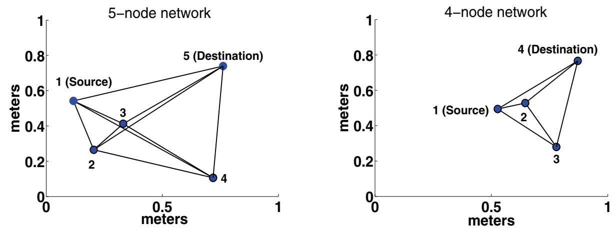

Fig. 2. Illustration of a sample network topology

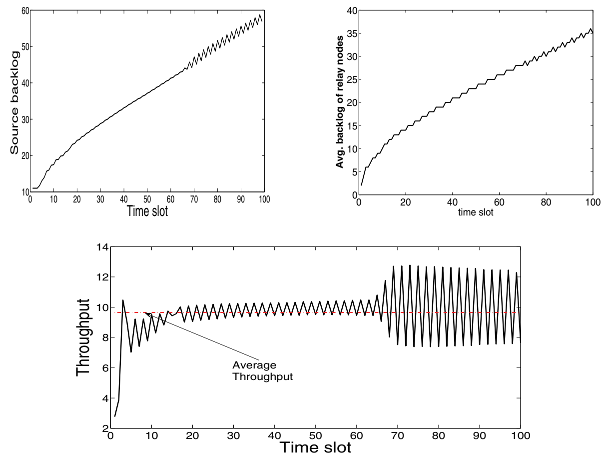

Fig. 3. Performance of BPPC for a 5-node wireless network with λ = 11

$\mathbf{\Pi_2}$ into a sequence of simpler conditional problems, each of which either has a closed-form solution or can be formulated as a simple convex optimization problem. This reduces the complexity of the extended WMMSE algorithm as compared to successive approximation. Moreover, using successive approximation we have to solve a series of approximations of $\mathbf{\Pi_2}$, whereas the extended WMMSE algorithm directly aims for the solution of $\mathbf{\Pi_2}$, even though the global optimum cannot be guaranteed due to NP-hardness. The complexity per cycle of updates for the Extended WMMSE algorithm is dominated by $\mathbf{v}$-update and $\mathbf{c}$-update steps. The optimization for the $\mathbf{v}$-update involves a quadratic objective with second-order cone constraints and its worst-case complexity [26] is $\mathcal{O}\left(L^4\right)$, where $L = |\mathcal{L}|$. On the other hand, the worst case complexity for the $\mathbf{c}$-update [20] is $\mathcal{O}\left(L^2\right)$.

Both the successive approximation BPPC-IC and the Extended WMMSE algorithm converge monotonically in terms of their objective function. The objective function of the Extended WMMSE formulation $\mathbf{\Pi_5}$ for the relaxed problem is convex with respect to each block of variables, and differentiable. Furthermore, the constraints associated to each block of variables are convex. Hence, the block coordinate descent algorithm converges to a stationary point of the optimization problem $\mathbf{\Pi_5}$ which in turn is a stationary point of $\mathbf{\Pi_2}$ [29].

V. Simulation Results

In this section, simulation results will be presented for the BPPC-IC problem solved approximately using successive approximation, and the extended WMMSE algorithm. The BPPC-IC policy is compared with BPPC [13] and BPPC-RIC2 policies (IV-A2). The comparison is based on maximum stable throughput, and source and relay node backlogs. The simulation parameters are specified in Table I.

The channel is assumed to have only propagation path loss. If $d_{\ell k}$ is the distance between Rx of link ℓ and Tx of link k and $G_{\ell k}$ the corresponding channel gain, then $G_{\ell k} = (d_{\ell k})^{-\alpha}$ where α is the path loss exponent. A deterministic and periodic input traffic of λ packets per slot into the source node is assumed for simulation.

Fig. 2 illustrates sample 4-node and 5-node network topologies used for simulation. The nodes are denoted by solid circles and the solid lines denote the bidirectional link between node pairs. The throughput, source and relay node backlogs have been plotted in Figs. 3-5 for N = 5, λ = 11 packets per slot and T = 30 dB. The source and relay node backlogs increase with time for the BPPC policy, whereas they remain bounded for BPPC-IC and BPPC-RIC2 policies. This highlights the ability of interference cancellation to stabilize the network and handle more traffic, thereby increasing its maximum stable throughput. Furthermore, the average source node backlogs for BPPC-IC are lesser than for BPPC-RIC2.

TABLE I
Simulation Parameters

Symbol	Description	Value
N	Number of nodes	4 / 5
P	Max. power per link	5 W
σ^2	Noise variance	$10^{-12}\,W$
G_{sg}	Spreading / beamforming / coding gain	128
T	SINR threshold for decoding	10-30dB
ε	Tolerance parameter	0.1
α	Path loss exponent	4

For BPPC-IC and BPPC-RIC2, when interference cancellation is (de-)activated on a link, it results in a significant change in the SINR (and capacity) of that link. Since the BPPC-IC problem is solved in each transmission scheduling round, and the interference cancellation activation pattern is generally different from one round to the next (as it takes into account queue backlogs that vary dynamically), these changes imply a higher variation in instantaneous throughput compared to BPPC.

Figs. 6-7 highlight the stabilization property of the BPPC-IC policy. The simulation for N = 4, λ = 9 packets per slot, T = 30 dB and an overall duration of 50 time slots is shown in Fig. 6. The network uses the BPPC policy for the first 25 slots and switches to the BPPC-IC policy for the next 25 slots. When the BPPC policy is used, the network is unstable and the node backlogs increase with time. However, the introduction of the BPPC-IC policy stabilizes the network and bounds the backlogs at source and relay nodes. It is also noteworthy that the average throughput also increases once the BPPC-IC policy is employed.

Fig. 7 compares the BPPC-IC and BPPC-RIC2 policies in terms of their ability to stabilize a network. Here we simulate a 5-node network with λ = 12 packets per slot and T = 30 dB for 100 time slots. For the first 50 slots, the network uses the BPPC policy, which is followed by the BPPC-IC policy in one case and the BPPC-RIC2 policy in the other, for the subsequent 50 time slots. The variation of source node backlogs with time is plotted in Fig. 7. Even

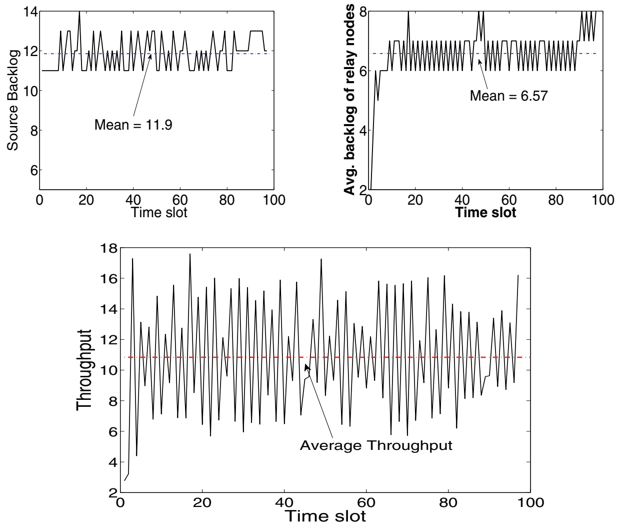

Fig. 4. Performance of BPPC-IC policy for a 5-node wireless network with λ = 11 and T = 30dB

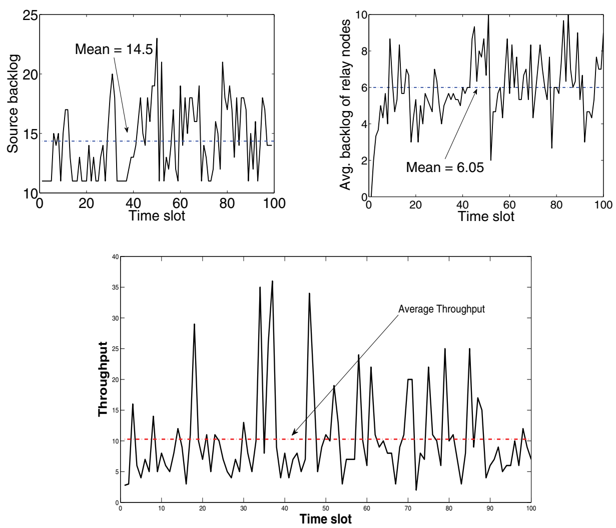

Fig. 5. Performance of BPPC-RIC2 policy for a 5-node wireless network with λ = 11 and T = 30dB

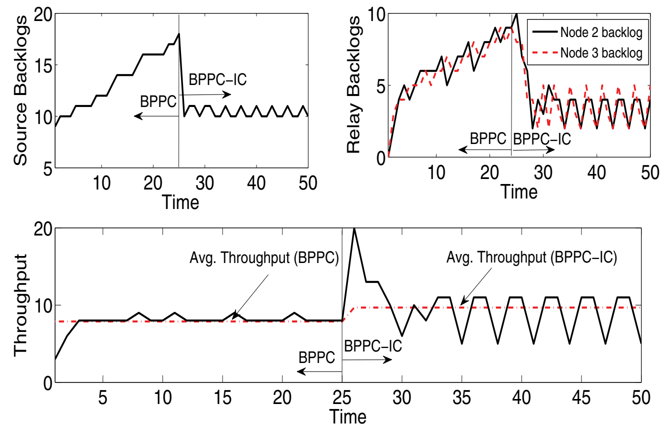

Fig. 6. Network stability and throughput performance of BPPC and BPPC-IC policies in a 4-node network for λ = 9 and T = 30dB

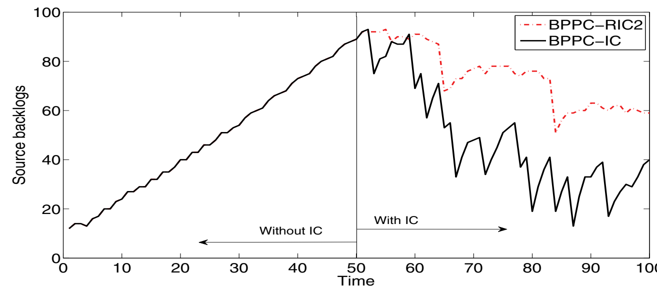

Fig. 7. Comparison of source node backlog stability of BPPC-IC and BPPC-RIC2 policies in a wireless multi-hop network with 5 nodes for λ = 12 and T = 30dB

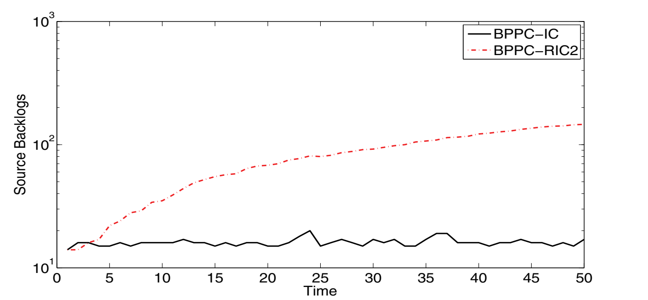

Fig. 8. Comparison of source backlogs for BPPC-IC and BPPC-RIC2 policies for 5-node wireless network with λ = 14 and T = 20dB

though both BPPC-IC and BPPC-RIC2 stabilize the network, the average source node backlog for BPPC-RIC2 is higher than that for BPPC-IC, indicating higher congestion (and thus higher average delay, by Little's Theorem) when the former is employed.

The maximum stable throughput is compared for various policies in Table II. It can be observed that there is a significant increase in the maximum stable throughput (up to 42.8% for the 4-node network, and 75% for the 5-node network) when interference cancellation is employed. Notice that the % increase in maximum stable throughput is higher for the larger network, so higher gains can be envisioned for larger networks, albeit this comes at the cost of higher complexity of solving the BPPC-IC problem.

Fig. 8 compares the source node backlogs of the BPPC-IC and BPPC-RIC2 policies for N=5, $\lambda = 14$ packets per slot and T = 20 dB. It is clear that the average source node backlog is higher for the network using the BPPC-RIC2 policy. Also, it can be seen from the figure that the source node backlog increases with time for the BPPC-RIC2 policy, whereas it remains bounded for the BPPC-IC policy.

Fig. 9 compares the approximate solutions of the BPPC-IC problem obtained from the extended WMMSE algorithm and the successive approximation algorithm. First, a problem instance of the BPPC-IC problem is simulated for $N = 5$, $\lambda = 11$ packets per slot and T = 10 dB using both algorithms. The throughput obtained from the simulation mentioned above for the extended WMMSE algorithm and the successive approximation algorithm is plotted on top. Even though the values for each time slot are different, the average throughput is almost the same in both cases. The bottom plot in Fig. 9 compares the simulation time for solving 100 BPPC-IC problem instantiations, using extended WMMSE and succes-

TABLE II
MAXIMUM STABLE THROUGHPUT COMPARISON ($T = 30dB$)

IC policy	4-node network	5-node network
BPPC	7	8
BPPC-RIC2	9	12
BPPC-IC	10	14

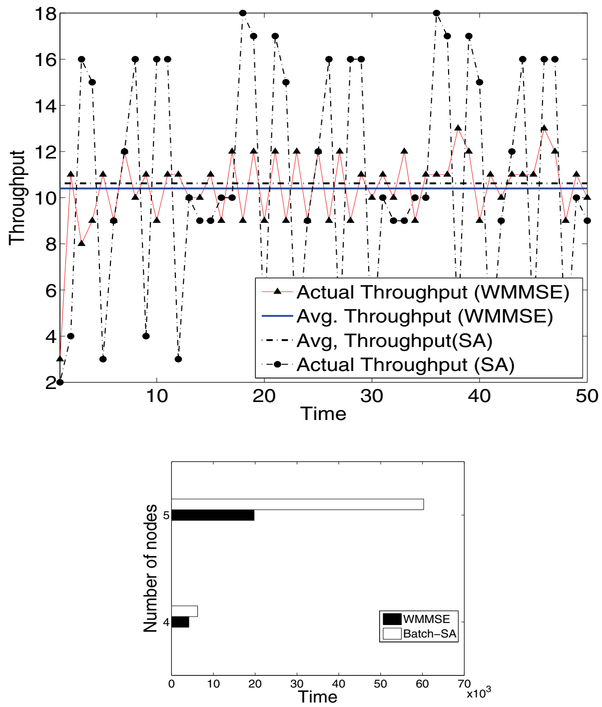

Fig. 9. Comparison of throughput performance of BPPC-IC using Successive approximation and WMMSE algorithm solutions for BPPC-IC problem with N = 5 and λ = 11 with T = 10dB

sive approximation. It is clear that the extended WMMSE algorithm takes significantly less time than its counterpart.

From the plots, it is clear that interference cancellation enhances the maximum stable throughput and reduces congestion (and therefore average delay) in the network. Furthermore, among the two different IC policies considered, BPPC-IC has higher maximum stable throughput and smaller average backlogs / delay than the BPPC-RIC2 policy.

VI. CONCLUSIONS

Interference cancellation is a well-appreciated tool in the context of multiuser detection at the physical layer, and in multiuser information theory. The starting point and premise of this paper is that interference cancellation can be equally valuable at the network layer: when judiciously combined with power control it can boost throughput and reduce delay *at the network layer*. This can be accomplished by extending the back-pressure power control framework to explicitly account for interference cancelation. The joint BPPC-IC problem is NP-hard, but two approximations were developed, based on successive geometric programming approximation, and an extended weighted minimum mean squared error (WMMSE) reformulation. The latter is typically much faster than the former, and the two usually yield similar throughput and delay performance. The pointwise solutions can differ though, and sometimes one comes out better than the other; there is no clear winner in terms of throughput / delay performance, but the extended WMMSE approach will likely prevail due to reduced complexity.

Beyond the specific approximations and algorithms, however, the main take-home message is clear: when interference cancellation is properly combined with power control in a cross-layer framework, it does yield significant gains in throughput and average delay (due to reduced backlogs, per Little's Theorem [28]) at the network layer. This motivates the pursuit of simpler engineering solutions that will manage to reap a good part of the benefit of interference cancellation at a small fraction of the complexity associated with convex optimization methods. Another practical issue to be dealt with is (non-)availability of global channel state information (CSI), and proper mechanisms for local CSI dissemination.

VII. APPENDIX I

Define

$$\begin{aligned}\psi_1(\mathbf{v},\mathbf{c}) &= \sum_{\ell=1}^{L} D_\ell(t)\left[\sum_{\substack{m=0\\m\neq\ell}}^{L}\log\left(1+c_{\ell m}\gamma_{\ell m}\right)\right]\\ &= \sum_{\ell=1}^{L}\sum_{\substack{m=0\\m\neq\ell}}^{L} D_\ell(t)\log\left(1+\frac{c_{\ell m}G_{\ell\ell}{v_\ell}^2}{\frac{1}{G_{sg}}\sum_{\substack{k=1\\k\neq\ell,m}}^{L}G_{\ell k}{v_k}^2+\sigma_\ell^2}\right)\\ &= \sum_{\ell=1}^{L} D_\ell(t)\left[\sum_{\substack{m=0\\m\neq\ell}}^{L}\log\left(e_{\ell m}(u^*_{\ell m},c_{\ell m},\mathbf{v})\right)\right] \qquad (29)\end{aligned}$$

This is the objective function of $\mathbf{\Pi_2}$ with p_ℓ replaced by ${v_\ell}^2$. The last equality is obtained from (20). Also define

$$\psi_2(\mathbf{u},\mathbf{w},\mathbf{v},\mathbf{c}) = \sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{L}_{\ell-}} D_\ell(t)\Big(w_{\ell m}\, e_{\ell m}(u_{\ell m},c_{\ell m},\mathbf{v}) - \log w_{\ell m}\Big) \qquad (30)$$

Let $(\mathbf{u}^*,\mathbf{w}^*,\mathbf{v}^*,\mathbf{c}^*)$ be a stationary point of $\mathbf{\Pi_5}$. Therefore from KKT conditions, we have

$$\begin{aligned}&\frac{\partial\left(\psi_2(\mathbf{u}^*,\mathbf{w}^*,\mathbf{v}^*,\mathbf{c}^*)\right)}{\partial u_{\ell m}}(u_{\ell m}-u^*_{\ell m})\leq 0,\\ &\frac{\partial\left(\psi_2(\mathbf{u}^*,\mathbf{w}^*,\mathbf{v}^*,\mathbf{c}^*)\right)}{\partial w_{\ell m}}(w_{\ell m}-w^*_{\ell m})\leq 0,\\ &Tr\left(\nabla_{\mathbf{x}}\left(\psi_2(\mathbf{u}^*,\mathbf{w}^*,\mathbf{v}^*,\mathbf{c}^*)\right)^T(\mathbf{x}-\mathbf{x}^*)\right)\leq 0,\ \forall\mathbf{x}\in\mathcal{S}\end{aligned} \qquad (31)$$

$\forall m\in\mathcal{L}_{\ell-}$, $\forall\ell\in\mathcal{L}$ and $\mathbf{x}=[\mathbf{v}^T,\ \mathbf{vec}(\mathbf{c})^T]^T$, $\mathcal{S}$ is the feasible set for $\mathbf{\Pi_5}$.

Using chain rule, we get

$$\begin{aligned}\frac{\partial(\psi_2(\mathbf{u}^*,\mathbf{w}^*,\mathbf{v}^*,\mathbf{c}^*))}{\partial v_\ell} &= \sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{L}_{\ell-}} D_\ell(t)\, w^*_{\ell m}\frac{\partial(e^*_{\ell m})}{\partial v_\ell}\\ &= \sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{L}_{\ell-}} D_\ell(t)\,(e^*_{\ell m})^{-1}\frac{\partial(e^*_{\ell m})}{\partial v_\ell} = \frac{\partial(\psi_1(\mathbf{v}^*,\mathbf{c}^*))}{\partial v_\ell}\end{aligned} \qquad (32)$$

where $e^*_{\ell m} = e_{\ell m}(u^*_{\ell m}, c^*_{\ell m}, \mathbf{v}^*)$. The second equality is obtained from (19) and (20). Similarly, we also get

$$\frac{\partial(\psi_2(\mathbf{u}^*, \mathbf{w}^*, \mathbf{v}^*, \mathbf{c}^*))}{\partial c_{\ell m}} = \frac{\partial(\psi_1(\mathbf{v}^*, \mathbf{c}^*))}{\partial c_{\ell m}} \tag{33}$$

Hence from (31),(32) and (33), we get

$$Tr\left(\nabla_{\mathbf{x}}\left(\psi_2(\mathbf{u}^*, \mathbf{w}^*, \mathbf{v}^*, \mathbf{c}^*)\right)^T(\mathbf{x}-\mathbf{x}^*)\right) = Tr\left(\nabla_{\mathbf{x}}\left(\psi_1(\mathbf{v}^*, \mathbf{c}^*)\right)^T(\mathbf{x}-\mathbf{x}^*)\right),\ \forall \mathbf{x}\in\mathcal{S} \tag{34}$$

Therefore, if $(\mathbf{u}^*, \mathbf{w}^*, \mathbf{v}^*, \mathbf{c}^*)$ is a stationary point of $\mathbf{\Pi_5}$, it follows from (31) and (34) that

$$Tr\left(\nabla_{\mathbf{x}}\left(\psi_1(\mathbf{v}^*, \mathbf{c}^*)\right)^T(\mathbf{x}-\mathbf{x}^*)\right) \le 0,\ \forall \mathbf{x}\in\mathcal{S} \tag{35}$$

which is the optimality condition for $\mathbf{\Pi_2}$.

VIII. Appendix II

In the extended WMMSE algorithm, the Lagrangian for the optimization problem $\mathbf{\Pi_9}$ (28) is

$$L_c(\mathbf{c},\mu,\lambda) = \sum_{\ell=1}^{L}\sum_{m\in\mathcal{L}_{\ell-}} D_\ell \log(1+\gamma_{\ell m}c_{\ell m}) - \sum_{\ell\in\mathcal{L}}\mu_\ell \sum_{m\in\mathcal{L}_{\ell-}}(c_{\ell m}-1) - \sum_{\ell\in\mathcal{L}}\left(\sum_{m\in\mathcal{L}_{\ell-}\setminus\{0\}}\lambda_{\ell m}(Tc_{\ell m}-\Gamma_{\ell m})\right) \tag{36}$$

Here $\gamma_{\ell m}$ is the SINR at the receiver of the ℓ^{th} link after canceling out the data from the m^{th} link and $\Gamma_{\ell m}$ is the SINR at the receiver of the ℓ^{th} link while decoding the transmission from m^{th} link. Note that both $\gamma_{\ell m}$ and $\Gamma_{\ell m}$ are constants, since we fix $p_\ell = p^*_\ell$ (obtained from the $\mathbf{v}$-update), $\forall\,\ell\in\mathcal{L}$, (as described in the algorithm). In this section, we come up with conditions for the optimal cancellation coefficients in $\mathbf{\Pi_9}$ to be close to $\{0,1\}$, so that the approximate solution to the interval-relaxed problem is very close to the original problem (with binary constraints). From (36), equating the partial derivative of $L_c(\mathbf{c},\mu,\lambda)$ w.r.t. $c_{\ell m}$ to zero, we get,

For $m=0$,

$$\frac{D_\ell\gamma_{\ell 0}}{1+\gamma_{\ell 0}c^*_{\ell 0}} - \mu^*_\ell = 0 \tag{37}$$

For $m\neq 0$,

$$\frac{D_\ell\gamma_{\ell m}}{1+\gamma_{\ell m}c^*_{\ell m}} - \mu^*_\ell - \lambda^*_{\ell m}T = 0 \tag{38}$$

Substituting (37) into (38), we get

$$\lambda^*_{\ell m}T = D_\ell\left(\frac{\gamma_{\ell m}}{1+\gamma_{\ell m}c^*_{\ell m}} - \frac{\gamma_{\ell 0}}{1+\gamma_{\ell 0}c^*_{\ell 0}}\right) \tag{39}$$

Case 1: $c^*_{\ell 0} = 1-\varepsilon,\ c^*_{\ell m} = \tilde{\varepsilon} \le \varepsilon$

From (39), we have

$$\lambda^*_{\ell m}T = D_\ell\left(\frac{\gamma_{\ell m}}{1+\gamma_{\ell m}\tilde{\varepsilon}} - \frac{\gamma_{\ell 0}}{1+(1-\varepsilon)\gamma_{\ell 0}}\right) \tag{40}$$

From the complementary slackness condition for the interference cancellation constraints, we get

$$\lambda^*_{\ell m}\Gamma_{\ell m} = \lambda^*_{\ell m}Tc^*_{\ell m} \tag{41}$$

Using the fact that $c^*_{\ell m} = \tilde{\varepsilon}$ and modifying (40),

$$\lambda^*_{\ell m} = \frac{D_\ell}{\Gamma_{\ell m}}\left[\frac{\tilde{\varepsilon}}{\tilde{\varepsilon}+\frac{1}{\gamma_{\ell m}}} + \frac{\tilde{\varepsilon}}{\varepsilon - \frac{1+\gamma_{\ell 0}}{\gamma_{\ell 0}}}\right] \tag{42}$$

The term inside the bracket is an increasing function of $\tilde{\epsilon}$. Hence, from (40) and (42) and the fact that $\tilde{\epsilon}\le\epsilon$ we get,

$$\lambda^*_{\ell m} \le \frac{D_\ell}{\Gamma_{\ell m}}\left[\frac{\varepsilon}{\varepsilon+\frac{1}{\gamma_{\ell m}}} + \frac{\varepsilon}{\varepsilon - \frac{1+\gamma_{\ell 0}}{\gamma_{\ell 0}}}\right] \tag{43}$$

$$T \ge \frac{D_\ell}{g_\lambda(\varepsilon)}\left(\frac{\gamma_{\ell m}}{1+\gamma_{\ell m}\varepsilon} - \frac{\gamma_{\ell 0}}{1+(1-\varepsilon)\gamma_{\ell 0}}\right) \tag{44}$$

where $g_\lambda(\varepsilon) = \frac{D_\ell}{\Gamma_{\ell m}}\left[\frac{\varepsilon}{\varepsilon+\frac{1}{\gamma_{\ell m}}} + \frac{\varepsilon}{\varepsilon-\frac{1+\gamma_{\ell 0}}{\gamma_{\ell 0}}}\right]$. Now, $g_\lambda(\varepsilon)$ is an increasing function of ϵ, when $\frac{dg_\lambda(\varepsilon)}{d\varepsilon} > 0$.

$$\frac{dg_\lambda(\varepsilon)}{d\varepsilon} > 0 \Rightarrow \frac{\frac{1}{\gamma_{\ell m}}}{\left(\varepsilon+\frac{1}{\gamma_{\ell m}}\right)^2} > \frac{1+\frac{1}{\gamma_{\ell 0}}}{\left(\varepsilon-\left(1+\frac{1}{\gamma_{\ell 0}}\right)\right)^2} \tag{45}$$

$$\varepsilon^2\left(1+\frac{1}{\gamma_{\ell 0}}-\frac{1}{\gamma_{\ell m}}\right) + 4\varepsilon\left(1+\frac{1}{\gamma_{\ell 0}}\right)\frac{1}{\gamma_{\ell m}} - \left(1+\frac{1}{\gamma_{\ell 0}}-\frac{1}{\gamma_{\ell m}}\right)\left(1+\frac{1}{\gamma_{\ell 0}}\right)\frac{1}{\gamma_{\ell m}} < 0 \tag{46}$$

Simplifying the quadratic inequality in (45) and using the fact that $\epsilon > 0$ and $\gamma_{\ell m} > \gamma_{\ell 0}$, it can be shown that

$$\frac{dg_\lambda(\varepsilon)}{d\varepsilon} > 0,\quad \forall\varepsilon\in[0,\varepsilon_{up}]\text{, where}$$
$$\varepsilon_{up} = \left[\left(\sqrt{\left(1+\frac{1}{\gamma_{\ell 0}}\right)\frac{1}{\gamma_{\ell m}}}\right)\frac{\sqrt{1+\frac{1}{\gamma_{\ell 0}}}-\sqrt{\frac{1}{\gamma_{\ell m}}}}{\sqrt{1+\frac{1}{\gamma_{\ell 0}}}+\sqrt{\frac{1}{\gamma_{\ell m}}}}\right] \tag{47}$$

Since $\left(\frac{\gamma_{\ell m}}{1+\gamma_{\ell m}\varepsilon} - \frac{\gamma_{\ell 0}}{1+(1-\varepsilon)\gamma_{\ell 0}}\right)$ increases with decrease in ε, from (44) and (47), it can be seen that the lower bound for T increases with decreasing ε for $\varepsilon\in[0,\varepsilon_{up}]$. Thus

$$\boxed{\frac{c^*_{\ell 0}}{c^*_{\ell m}} = \frac{1-\varepsilon}{\tilde{\varepsilon}} \ge \frac{1}{\varepsilon}-1}\text{, if}$$
$$\boxed{T \ge \frac{D_\ell}{g_\lambda(\varepsilon)}\left(\frac{\gamma_{\ell m}}{1+\gamma_{\ell m}\varepsilon} - \frac{\gamma_{\ell 0}}{1+(1-\varepsilon)\gamma_{\ell 0}}\right)}\ \forall\varepsilon\in[0,\varepsilon_{up}] \tag{48}$$

Case 2: $c^*_{\ell m} = 1-\varepsilon,\ c^*_{\ell 0} = \tilde{\varepsilon} \le \varepsilon$

Define

$$\Gamma^{\max}_{\ell m} = \max_{\substack{k\in\mathcal{L}_{\ell-}\setminus\{0\}\\ T<\Gamma_{\ell k}}} \Gamma_{\ell k} \tag{49}$$

From (49), since $T < \Gamma^{\max}_{\ell}$, it is straightforward that the constraint $T\,c_{\ell m} \le \Gamma_{\ell m}$, will be met with strict inequality at optimality, as $\frac{\Gamma^{\max}_{\ell m}}{T} > 1$ and $c_{\ell m}\le 1$. Hence, from the complementary slackness condition, $\lambda^*_{\ell m} = 0$. Thus from (39), we get

$$\begin{aligned}
&\frac{\gamma_{\ell m}}{1+\gamma_{\ell m} c^*_{\ell m}} = \frac{\gamma_{\ell 0}}{1+\gamma_{\ell 0} c^*_{\ell 0}} \\
&c^*_{\ell m} - c^*_{\ell 0} = \frac{1}{\gamma_{\ell 0}} - \frac{1}{\gamma_{\ell m}} \\
&\Rightarrow (1-2\varepsilon) \le (1-\varepsilon-\tilde{\varepsilon}) \\
&= \frac{1}{G_{sg} G_{\ell\ell} p_\ell} \left(\sum_{\substack{k=1\\k\neq l}}^{L} G_{\ell k} p_k - \sum_{\substack{j=1\\j\neq l,m}}^{L} G_{\ell j} p_j \right) = \frac{G_{\ell m} p_m}{G_{sg} G_{\ell\ell} p_\ell}
\end{aligned} \tag{50}$$

Therefore, from (49) and (50), it can be seen that

$$\boxed{\frac{c^*_{\ell m}}{c^*_{\ell 0}} = \frac{1-\varepsilon}{\tilde{\varepsilon}} \ge \frac{1}{\varepsilon} - 1},$$

$$\text{if} \quad \boxed{T < \Gamma^{\max}_{\ell}, \quad \frac{p_m}{p_\ell} \ge (1-2\varepsilon) G_{sg} \frac{G_{\ell\ell}}{G_{\ell m}}} \tag{51}$$

Thus from (48) and (51), for $\varepsilon \in [0, \varepsilon_{up}]$ and $\frac{D_\ell}{g_\lambda(\varepsilon)} \left(\frac{\gamma_{\ell m}}{1+\gamma_{\ell m}\varepsilon} - \frac{\gamma_{\ell 0}}{1+(1-\varepsilon)\gamma_{\ell 0}} \right) \le T < \Gamma^{\max}_{\ell}$, it is possible to obtain near-binary cancellation coefficients as solution to $\mathbf{\Pi_9}$.

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Balasubramanian Gopalakrishnan received his B.Tech degree in Electronics and Telecommunication Engineering from University of Kerala, India in 2006, and his M.Tech degree in Electronics and Electrical Communication Engineering from Indian Institute of Technology, India in 2008. Since 2009, he has been working towards his Ph.D. degree at the Department of Electrical and Computer Engineering, University of Minnesota. His current research interests are signal processing and cross layer optimization techniques for wireless networks, and resource allocation in cognitive radios.

Nicholas D. Sidiropoulos (F'09) received the Diploma in Electrical Engineering from the Aristotelian University of Thessaloniki, Greece, and M.S. and Ph.D. degrees in Electrical Engineering from the University of Maryland - College Park, in 1988, 1990 and 1992, respectively. He served as Assistant Professor at the University of Virginia (1997-1999); Associate Professor at the University of Minnesota - Minneapolis (2000-2002); Professor at the Technical University of Crete, Greece (2002-2011); and Professor at the University of Minnesota - Minneapolis (2011-). His current research focuses primarily on signal and tensor analytics, with applications in cognitive radio, big data, and preference measurement. He received the NSF/CAREER award (1998), the IEEE Signal Processing Society (SPS) Best Paper Award (2001, 2007, 2011), and the IEEE SPS Meritorious Service Award (2010). He has served as IEEE SPS Distinguished Lecturer (2008-2009), and Chair of the IEEE Signal Processing for Communications and Networking Technical Committee (2007-2008).