Adaptive Control Design and Analysis
(**Gang Tao**, John Wiley & Son, 2003; ISBN 0-471-27452-6; TJ217.T34 2003)

Errata and Comments

- On page xx, line two, "Lennart Ljung, Wei Lin, Rogelio Lozano, David Mayne, Iven Mareels" should be "Wei Lin, Lennart Ljung, Rogelio Lozano, Iven Mareels, David Mayne".
- On page 30, the line after (1.138), "of (1.135)" should be "of (1.135):".
- On page 31, line 1, it should be "(i.e., $r(s) = \frac{1}{s}$)".
- On page 32, line 12, "efficiently" should be "effectively".
- On page 34, line 3, change "$a = -5$" to "$a_p = 5$".
- On page 37, in (2.10), change " $\Phi(t,t_0) = e^{A(t-t_0)} = \mathcal{L}^{-1}\left[(sI-A)^{-1}\right]$" to

$$\Phi(t,t_0) = e^{A(t-t_0)},\ e^{At} = \mathcal{L}^{-1}\left[(sI-A)^{-1}\right].$$

- On page 38, after (2.15), "$\int_{t_0}^{t} D(t)\delta(t-\tau)u(\tau)dt = D(t)u(t)$" should be "$\int_{t_0}^{t} D(t)\delta(t-\tau)u(\tau)d\tau = D(t)u(t)$".
- On page 40, below (2.24), it should be "$\det[P_l(s)] \not\equiv 0$ and $\det[P_r(s)] \not\equiv 0$".
- On page 43, in (2.36), "$G_0(s)$" should be "$G(s)$".
- On page 46, in (2.50), "k_1" and "k_2" should be "c_1" and "c_2".
- On page 47, line 2, "$A(t) \in R^{n\times n}$" should be "$A(t) \in R^{m\times n}$".
- On page 48, in (2.70), "$\|x(\cdot)\|_p = \left(\int_0^\infty \|x(t)\|_p^p dt\right)$" should be "$\|x(\cdot)\|_p = \left(\int_0^\infty \|x(t)\|_p^p dt\right)^{\frac{1}{p}}$".
- On page 48, in Minkovski inequality, "$b > a$" should be "$\sigma > \tau$".
- On page 54, to use the given choices of $\phi(r)$ for Definitions 2.17e, 2.19e and 2.20e, the property (ii) of the class-$\mathcal{K}$ functions should be replaced by (ii′) $\phi(r_1) \le \phi(r_2)$, $\forall r_1 \le r_2$ (i.e., $\phi(\cdot)$ is a monotonically increasing function but not strictly monotonically increasing), leading to a different definition of the class-$\mathcal{K}$ functions [351, page 107], [426, 1st edition (1978)].

  With the given (and popular in the literature) definition of the class-$\mathcal{K}$ functions (i.e., $\phi(\cdot)$ strictly monotonically increasing), the equivalence of Definition 2.17 and Definition 2.17e follows from the fact that there exists a class-$\mathcal{K}$ function $\phi(r)$ such that $\phi(r) \le \inf_{r\le\|x\|\le h} W(x)$ [179, 3rd edition (2003)], [426]. The equivalence of Definition 2.19 and Definition 2.19e follows from the fact that there exists a class-$\mathcal{KR}$ function $\phi(r)$ such that $\phi(r) \le \inf_{r\le\|x\|} W(x)$. The equivalence of Definition 2.20 and Definition 2.20e follows from that fact that there exists a class-$\mathcal{K}$ function $\phi(r)$ such that $\phi(r) \ge \sup_{\|x\|\le r} W(x)$. Such functions $\phi(r)$ can be constructed based on $\inf_{r\le\|x\|\le h} W(x)$, $\inf_{r\le\|x\|} W(x)$, and $\sup_{\|x\|\le r} W(x)$, respectively [259], [426].
- On page 56, line 18 and line 19, change "(ii) $\phi(r_1) < \phi(r_2)$, for any $0 \le r_1 < r_2 \le h$; and $\phi(\|x\|) \le W(x)$, $\forall x \in B(h)$" to "(ii) $\phi(r_1) \le \phi(r_2)$, for any $0 \le r_1 \le r_2 \le h$; and (iii) $\phi(\|x\|) \le W(x)$, $\forall x \in B(h)$".

- On page 57, line 7, "$\frac{dW(x(t)}{dt}$" should be "$\frac{dW(x(t))}{dt}$".
- On page 57, 5 lines above (2.84), add "}" to close up $B(h)$.
- On page 69, in (2.131), replace "=" by "$\leq$".
- On page 71, last line of (2.139), "$\leq e^{-\delta t}\|h_\delta(\cdot)\|_1 \int_0^t e^{\delta t}|x_2(\tau)|\,d\tau = \ldots$" should be

$$\text{“}\leq e^{-\delta t}\|h_\delta(\cdot)\|_1 \int_0^t e^{\delta\tau}|x_2(\tau)|\,d\tau = \ldots\text{”}.$$

- On page 73, line 1, "**Lemma 2.4**" should be "**Lemma 2.5**".
- page 78, under "Definition 2.25", "$h_s(s) = \frac{R+sC}{sCR}$" should be "$h_s(s) = \frac{RCs+1}{sC}$", and "$H_s(s) = \frac{R}{sC+R}$" should be "$H_s(s) = \frac{R}{sRC+1}$".
- On page 78, Lemma 2.10, (iii), the last part "or $\lim_{\omega\to\infty} \frac{h(j\omega)}{j\omega} > 0$ if $n^* = -1$" should be "or $\lim_{\omega\to\infty} \mathrm{Re}[h(j\omega)] > 0$ and $\lim_{\omega\to\infty} \frac{h(j\omega)}{j\omega} > 0$ if $n^* = -1$" (as given in [152]).

  Or, the last two parts " or $\lim_{\omega\to\infty} \mathrm{Re}[h(j\omega)] > 0$ if $n^* = 0$, or $\lim_{\omega\to\infty} \frac{h(j\omega)}{j\omega} > 0$ if $n^* = -1$" is replaced by "or $\lim_{\omega\to\infty} \mathrm{Re}[h(j\omega)] > 0$ and $\lim_{\omega\to\infty} \frac{h(j\omega)}{j\omega} > 0$ if $n^* = -1$"; that is, the case of $n^* = 0$ is excluded from (iii), as the result for $n^* = 0$ follows from (ii), by the continuity of $h(s)$ (for $h(s) = c(sI - A)^{-1}b + d$ with $d \neq 0$, if $\mathrm{Re}[h(j\omega)] > 0$ for all $\omega \in (-\infty, +\infty)$, then $d > 0$).
- On page 79, below (2.176), "$(A + \delta_0 I, q)$ is observable" should be "$(A + \delta_0 I, q^T)$ is observable".

  Remark: If $A + \delta_0 I$ is stable, then (2.177) has a unique solution $P = P^T \geq 0$, and if, in addition, $(A + \delta_0 I, q^T)$ is observable, then (2.177) has a unique solution $P = P^T > 0$.
- On page 80, below (2.178), "an absolutely integrable function $f(t)$ is also integrable because both the negative and positive parts of such an $f(t)$ are integrable" should be "an absolutely integrable function $g(t)$ is also integrable because both the negative and positive parts of such a $g(t)$ are integrable, and the integral of $g(t)$ is the sum of the two integrals".
- On page 82, line 10, some more details are:

  From $g(t) - g(0) = \int_0^t \dot{g}(\tau)d\tau \leq \int_0^t |\dot{g}(\tau)|d\tau < \infty$ for any $t \geq 0$, we have that $\int_0^t |\dot{g}(\tau)|d\tau$ has a limit, and so does $\int_0^t \dot{g}(\tau)d\tau$, which means that $g(t) = g(0) + \int_0^t \dot{g}(\tau)d\tau$ has a limit. Since $g(t) \in L^1$, the limit of $g(t)$ is zero.

  This proof has shown: in general, if $g(t) \in L^1$ and $\dot{g}(t) \in L^1$, then $\lim_{t\to\infty} g(t) = 0$.
- On page 83, in (2.187), "$f(0)$" should be "$g(0)$".
- On page 83, in Lemma 2.19, "$\lim_{t\to\infty} f(t) = 0$" should be "$\lim_{t\to\infty} g(t) = 0$".
- On page 85, equation (2.200) should be

$$\Phi(t, t_0) = A^{t-t_0} = \underbrace{AA\cdots A}_{i-i_0},\ t \geq t_0.$$

- On page 96, in Problem 2.7, change "uniformly" to "(uniformly)" (as only the boundedness is essential).

- On page 100, in (3.7), "$z(t) = y(t) = \theta^{*T}\phi(t)$" should be "$y(t) = \theta^{*T}\phi(t)$".
- On page 101, above (3.15), "$\omega_1(t) \in R^n$, $\omega_1(t) \in R^n$" should be "$\omega_1(t) \in R^n$, $\omega_2(t) \in R^n$".
- On page 103, for (3.24), $m(t) = \sqrt{\kappa + \phi^T(t)\phi(t)}$ with $\kappa > 0$ is also a choice.
- On page 104, for (3.30) and (3.31), $m(t) = \sqrt{\kappa + \phi^T(t)\phi(t)}$ with $\kappa > 0$ is also a choice, as the derivations on pages 105-107 are not explicitly dependent on on $m(t)$.
- On page 106, in line 6, delete "$\geq$" from "$z^T P(t_0)z \geq$".
- On page 107, in (3.45), the choice of the normalizing term $m(t) > 0$ is arbitrary at this point, while the choice of $m(t)$ in (3.32) is a desired one.
- On page 109, in line 11 (Theorem 3.1), change "C^n" to "C^{n+m+1}".
- On page 109, in line 26 (Proposition 3.1), change "C^n" to "C^{n+m+1}".
- On page 110, in line 5, "Then, we can define $\varepsilon(t) = \theta(t)\phi(t)$" should be "Then, we can define $\varepsilon(t) = \theta(t)\phi(t) - \bar{y}(t)$".
- On page 110, in Lemma 3.3, change "α_0" to "α_s" (as a generic number, different from α_0 in (3.52)).
- On page 111, at the end of the proof of Lemma 3.3, add "$\alpha_s = \sqrt{1 - \lambda_s}$".
- On page 116, in line 7, change "$\frac{\varepsilon(t)}{m(t)} \in L^2 \cap L^\infty$, and $\theta(t+1) - \theta(t) \in L^2$" to "$\frac{\varepsilon(t)}{m(t)} \in L^\infty$".
- On page 118, in (3.96), "$1 + \kappa\|P_s(t-1)\phi(t)\|_2^2$" should be "$\kappa + \|P_s(t-1)\phi(t)\|_2^2$".
- On page 119, below (3.104), to obtain (3.88) from (3.90), we used the matrix inversion formula:

  $$(A+BC)^{-1} = A^{-1} - A^{-1}B(I + CA^{-1}B)^{-1}CA^{-1},$$

  with $A = P^{-1}(t-1)$, $B = \frac{\phi(t)}{\kappa}$ and $C = \phi^T(t)$.
- On page 120, in line 17 (Theorem 3.2), change "C^n" to "C^{n+m+1}".
- On page 120, in line 27 (Proposition 3.2), change "C^n" to "C^{n+m+1}".
- On page 123, in (3.123), "$P(t_{0-1}) = \rho_0 I$" should be "$P(t_0 - 1) = \rho_0 I$".
- On page 132, in (3.178), "$\tilde{\theta}^T(t)\Gamma f(t)$" should be "$\tilde{\theta}^T(t)\Gamma^{-1} f(t)$".
- On page 133, below (3.182), "$i = 1, 2, \ldots, n_\theta$" should be "$j = 1, 2, \ldots, n_\theta$".
- On page 155, in line 1 and line 2, change "(motor voltage)" by "(motor torque generated by armature current)" and change "joint force" to "joint torque".
- On page 156, Assumption (A4.4) implies Assumption (A4.3) (because, with $P(s) = \det(sI - A)$, any possible pole-zero cancellation in $G(s)$ is stable if $Z(s)$ is stable).

  In (4.44), the closed-loop poles are the zeros of $P_m(s)$ and $Z(s)$, so that $Z(s)$ has to be stable, which implies that (A, B, C) needs to be stabilizable and detectable.

- On page 157, in (4.44), "$P_m(s)Z(s)\frac{1}{k_p}$" should be "$P_m(s)Z(s)$".
- On page 160, after (4.68), "$k_{3j}^* = d_j$" to "$k_{3j}^* = -d_j$".
- On page 161, for Section 4.3.2, the vectors B and B_d in (4.79) are not assumed to be parallel to each other, i.e., they can be independent.
- On page 164, for (4.93), the coefficients of cos terms are in general not zero (and thus need to be estimated for implementing the controller (4.92)) even if the coefficients of the cos terms in (4.90) and (4.91) are zero. If both d_{u0} and d_{y0} in (4.90) and (4.91) are zero, then k_{30}^* in (4.93) is zero and its estimate $k_{30}(t)$ in (4.92) can be set to be zero.
- On page 165, in (4.100), the time variable "t" is missing in those sin and cos signals.
- On page 170, in Lemma 4.2, "(4.93) and (4.94)" should be "(4.130) and (4.131)".

  In line 170, "using an estimate of $k_1^{*T}x(t)$ which needs" should be "using an estimate of $k_1^{*T}x(t)$, which needs".
- On page 172, below (4.143), "$L_r y(0) - x_2(0)$" should be "$x_2(0) - L_r y(0)$".
- On page 175, equation (4.159) should be

$$\rho^* = b_p,\ \theta^* = [k_1^*, k_2^*]^T,\ \theta = [k_1, k_2]^T$$

- On page 178, Assumption (A4.4-D) implies Assumption (A4.3-D).
- On page 185, in (4.222), the time variable "t" is missing in those sin and cos signals.
- On page 188, in line 6, "Parametrizations of State Feedback" should be "Parametrization of Observer-Based Feedback".

  In line 15, "using an estimate of $k_1^{*T}x(t)$ which" should be "using an estimate of $k_1^{*T}x(t)$, which".
- On page 200, line 1, if the original plant is described by: $P(s)C(s)[y](t) = k_p Z(s)C(s)[u](t)$, for some polynomial $C(s)$, we can still obtain (5.28) but (5.29) should be operated by $C(s)$. Then, to reach (5.30), $C(s)$ needs to be stable. Hence, for MRAC, the original state-space form (A_0, B_0, C_0) of (5.1) needs to be stabilizable and detectable.
- On page 204, at the end of Example 5.1, "$\alpha = 2$" should be "$\alpha = 1$".
- On page 207, above (5.69), "(5.681)" should be "(5.68)"; in (5.72), the parameter "a" is different from the polynomial vector "$a(s)$" in (5.6)–(5.7) on page 197.
- On page 215, line 10, "(5.119)" should be "(5.118)".
- On page 217, in (5.140), "$m_0(t)$" should be "$m(t)$".
- On page 233, the first inequality of equation (5.207) should be

$$\int_{t_1}^{t_2} e^2(t)\,dt \le \frac{1}{a_m}(V(e_p(t_1)) - V(e_p(t_2))) + \int_{t_1}^{t_2} \frac{\bar{d}^2(t)}{a_m^2}\,dt$$

- On page 237, below (5.222), "Using (5.83)" should "Using (5.34), (5.83)".
  From (5.34): $u(t) = \theta_1^T(t)\omega_1(t) + \theta_2^T(t)\omega_2(t) + \theta_{20}(t)y(t) + \theta_3(t)r(t) = \theta^T(t)\omega(t)$, we have

  $$|u| \leq T_{11}(s)[|z_0|](t) + T_{12}(s)[|y|](t) + c_{11}|y(t)| + c_{12}$$

  for some stable operators $T_{11}(s)$ proper and $T_{12}(s)$ strictly proper, and constants $c_{11} > 0$ and $c_{12} > 0$. Then, $m(t)$ in (5.222), with (5.82): $z_0(t) = T_1(s,\cdot)[y](t) + b_0(t)$, satisfies:

  $$m(t) \leq T_{13}(s)[|z_0|](t) + T_{14}(s)[|y|](t) + c_{13} \leq T_{15}(s)[|y|](t) + c_{14}$$

  for some stable, strictly proper operators $T_{13}(s)$, $T_{14}(s)$, $T_{15}(s)$, and constants $c_{13} > 0$, $c_{41} > 0$.

- On page 238, below (5.226), (5.34) and (5.192) (without $H(s)[d](t)$) should be cited.
  From (5.192) (without $H(s)[d](t)$): $e(t) = \frac{k_p}{P_m(s)}[\tilde{\theta}^T\omega](t) + \mu\Delta(s)[u](t)$, we have

  $$\begin{aligned} y(t) &= y_m(t) + \frac{k_p}{P_m(s)}[u(t) - \theta^{*T}\omega](t) + \mu\Delta(s)[u](t) \\ &= y_m(t) + \frac{k_p}{P_m(s)}[u](t) - \theta^{*T}\frac{k_p}{P_m(s)}[\omega](t) + \mu\Delta(s)[u](t), \end{aligned}$$

  from which it follows that

  $$|y(t)| \leq T_{21}(s)[|z_0|](t) + T_{22}(s)[|z|](t) + \mu T_{23}(s)|z_0|](t) + c_{21}$$

  for some stable and proper operators $T_{21}(s)$ and $T_{22}(s)$ (both strictly proper if $n^* > 1$) and stable and proper operator $T_{23}(s)$, and some constant $c_{21} > 0$.
  Then, $m(t)$ in (5.222) or (5.226), with (5.34) (leading to $|u| \leq T_{11}(s)[|z_0|](t) + T_{12}(s)[|y|](t) + c_{11}|y(t)| + c_{12}$) and (5.137) based on (5.225): $z_0(t) = T_1(s,\cdot)[z](t) + b_0(t)$, satisfies:

  $$m(t) \leq \frac{\delta_1}{s+\delta_0}[|u| + |y| + 1](t) \leq T_{24}(s)[|z|](t) + c_{22}$$

  for some stable and strictly proper operator $T_{24}(s)$, and constant $c_{22} > 0$.

- On page 241, before (5.244), "(5.223)" should be "(5.233)".
  In (5.247), $\pi(z(t))$ should be

  $$\begin{aligned} \pi(z(t)) = V(e(0), \tilde{\theta}(0)) &+ 2z(t)(2b_p \sin z(t) - |b_p|) + 4b_p \cos z(t) - 2b_p z^2(t)\cos z(t) \\ &- 2z_0(2b_p \sin z_0 - |b_p|) - 4b_p \cos z_0 + 2b_p z_0^2 \cos z_0 \end{aligned}$$

- On page 263, (5.344) holds, because, for $t \geq T = \max\{T_1, T_2\}$ (in fact, $-\frac{1}{a}\ln\left(\frac{a\delta}{2x_0}\right) > 0$ for δ small (as required to show $w(t)$ going to zero)), we have

  $$\begin{aligned} &\frac{x_0}{a}e^{-at}\left(e^{aT_1} - 1\right) \\ &\leq \frac{x_0}{a}e^{-a\left(T_1 - \frac{1}{a}\ln\left(\frac{a\delta}{2x_0}\right)\right)}\left(e^{aT_1} - 1\right) = \frac{x_0}{a}e^{-aT_1}e^{\ln\left(\frac{a\delta}{2x_0}\right)}\left(e^{aT_1} - 1\right) = \frac{x_0}{a}\frac{a\delta}{2x_0}\left(1 - e^{-aT_1}\right) \\ &= \frac{\delta}{2}\left(1 - e^{-aT_1}\right) \leq \frac{\delta}{2}. \end{aligned}$$

- On page 263, 5 lines below (5.344), "However, with (5.342), it has not been concluded that" should be "Moreover, with (5.342) and (5.159), since $\lim_{t\to\infty} d(t) = 0$, for this nominal case, it can be concluded that $\lim_{t\to\infty}(y(t) - y_m(t)) = 0$. However, for adaptive control with parameter estimates $\theta_i(t)$, it has not been concluded, due to the lack of the desired property: $\frac{\varepsilon(t)}{m(t)} \in L^2$ (or, more generally, $\frac{\varepsilon(t)}{m(t)} \in L^{1+\bar{\alpha}}$, see Section 5.4.2), that".

- On page 264, lines 5 and 6, "$i = 1, 2 \ldots, l$" should be "$i = 1, 2, \ldots, l$".

- On page 265, line 8, "part (i)" should be "part (iii)".

- On page 274, after (6.15), "$\bar{\theta*}$" should be "$\bar{\theta}^*$".

- On page 278, line 8, "adaptive law for $\theta(t)$" should be "adaptive laws for $\theta(t)$ and $\rho(t)$".

- On page 279, in Theorem 6.2, "$W_m(z) = z^{n^*}$" should be "$W_m(z) = z^{-n^*}$".

- On page 279, in (6.55), "z^*" should be "$k_p z^*$".

- On page 279, in (6.57), "$Z(z)$" should be "$k_p Z(z)$".

- On page 288, equation (6.121) should be

$$\begin{aligned}\phi(t) = [&u(t-n^*), \ldots, u(t-n^*-n+1),\\ &y(t-n^*), \ldots, y(t-n^*-n+1)]^T.\end{aligned}$$

- On page 311, after the last sentence, add "In particular, (5.137) is to be used to derive a counterpart of (5.141) based on which the closed-loop signal boundedness can be concluded. To establish (5.137), the condition that all zeros of $Z(s)$ are stable is crucial (see (5.135) where $G_0^{-1}(s) = \frac{P(s)}{k_p Z(s)}$ is present), when $\theta(t)$ is bounded."

- On page 316, (7.146) should be

$$\psi_1 = -\begin{bmatrix} \psi_c & 0 & \cdots & 0 & 0 \\ 1 & \psi_c & \cdots & \vdots & 0 \\ 0 & 1 & \cdots & 0 & \vdots \\ \vdots & 0 & \cdots & 0 & 0 \\ 0 & \vdots & \cdots & 0 & 0 \\ 0 & 0 & \cdots & \psi_c & 0 \\ 0 & 0 & \cdots & 1 & \psi_c \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ \vdots \\ q_{n_q-1} \\ 1 \end{bmatrix} + \begin{bmatrix} \lambda_0^c \\ \lambda_1^c \\ \vdots \\ \lambda_{n_q+n-2}^c \end{bmatrix}$$

  and (7.147) should be

$$\psi_2 = -\begin{bmatrix} d_0 \\ d_1 \\ \vdots \\ d_{n_q+n-2} \end{bmatrix} + d_{n_q+n-1} \begin{bmatrix} \lambda_0^c \\ \lambda_1^c \\ \vdots \\ \lambda_{n_q+n-3}^c \\ \lambda_{n_q+n-2}^c \end{bmatrix}.$$

- On page 319, above (7.165), "$\theta_p^* \in R^{n+m+1}$" should be "$\theta_p^* \in R^{2n}$".
- On page 319, in (7.166), "$\theta(0)$" should be "$\theta_p(0)$".
- On page 320, equation (7.172) should be

$$\hat{\theta}_{qp}(t) = [1, \hat{\rho}_{n+n_q-1}, \hat{\rho}_{n+n_q-2}, \ldots, \hat{\rho}_1, \hat{\rho}_0]^T \in R^{n+n_q+1},$$

  equation (7.173) should be

$$\hat{\theta}_z(t) = [0, \hat{z}_{n-1}, \hat{z}_{n-2}, \ldots, \hat{z}_1, \hat{z}_0]^T \in R^{n+1},$$

  equation (7.174) should be

$$\hat{\theta}_c(t) = [1, \hat{c}_{n-2}, \ldots, \hat{c}_1, \hat{c}_0]^T \in R^n,$$

  and equation (7.175) should be

$$\hat{\theta}_d(t) = [\hat{d}_{n+n_q-1}, \hat{d}_{n+n_q-2}, \ldots, \hat{d}_1, \hat{d}_0]^T \in R^{n+n_q}.$$

- One page 330, in (7.227), "$k_1(t) = \frac{-\hat{a}_p(t)-a_m}{\hat{b}_p(t)}$" should be "$k_1(t) = \frac{-\hat{a}_p(t)+a_m}{\hat{b}_p(t)}$".
- One page 332, below (7.245), "desired properties of (1.139)" should be "desired equation (1.115)".
- On page 333, in (7.253), "$\theta(0)$" should be "$\theta_p(0)$".
- On page 338, in (7.294), "$\theta(0)$" should be "$\theta_p(0)$".
- On page 346, in line 4, 8, 12, 21, 22, 25, "$p_d(D)$" should be "$P_d(D)$".
- On page 348, in part (iv) of Problem 7.11, change "Problem 5.1" to "Problem 5.3".
- On page 367, above (8.109), "$\hat{\beta}$ is the estimate of β" should be "where $\hat{\beta}_1$ is the estimate of β_1".
- On page 368, in (8.120), "$-\gamma_0$" should be "γ_0".
- On page 369, in (8.125), "$\dot{V}_3(t)$" should be "$\dot{V}_3$", and "$c_5 z_3 x_4^4$" should "$c_5 z_3^2 x_4^4$".
- On page 376, above (9.31), "$\Phi_{2i}^T(t)$" should be "$\Phi_{2i}(t)$", and "$\Phi_2^T(t)$" should be "$\Phi_2(t)$".
- On page 376 and page 377, in (9.31), "$\Phi_{2i}^T(t)$" should be "$\Phi_{2i}(t)$", $i = 1, 2, \ldots, M$.
- On page 377, in (9.36), "$2e^T(t)Pe(t)$" should be "$2e^T(t)PA_m e(t)$".
- On page 378, line 3, "$y(t)$" should be "$x(t)$".
- On page 379, before the second paragraph, add

  So far, motivated from adaptive control, we have derived the dynamic equation (9.39) which is to be used for the design of an adaptive estimator (identifier) of a plant

$$\dot{x} = Ax(t) + Bu(t) \tag{1}$$

  whose parameters A and B are to be estimated (identified). For (9.39), the matrix A_m now can be chosen independent of the matching condition (9.3): $A + BK_1^{*T} = A_m$, and its state vector

$x_m(t)$ is generated based on the measured plant state $x(t)$ and chosen input $u(t)$. Note that from now on $r(t)$, $K_1(t)$ and $K_2(t)$ all become irrelevant anymore. Next we will use $x_m(t)$ from (9.39) to construct an adaptive parameter estimation (system identification) scheme to generate the desired estimates $\hat{A}(t)$ and $\hat{B}(t)$ of the unknown system matrices A and B. In this case, (9.39) may be called an estimator or identifier state equation.

- On page 380, in (9.50), "$r(t)$" should be "$u(t)$"; line 19, "$y_m(t)$" should be "$x_m(t)$"; line 20, "$e(t) = x(t) - x_m(t)$" should be "$e(t) = x_m(t) - x(t)$".

- On page 383, three lines above (9.58), "$Z_r(D)$" should be "$Z_l(D)$";

- On page 386, line 3, change "polynomials divisible by s" to "polynomials divisible by D".

- On page 390, in the proof of Lemma 9.3, for (9.83), "$\partial_{ci}[R_l(D)] < \partial_{ci}[P_l(D)] \leq \nu$" as $P_l(D)$ divides from the right.

  To determine the degree of $\Theta_1^{*T}\bar{A}(D)$, we start (9.85) with

$$\Theta_1^{*T}\bar{A}(D) = \Lambda(D)I_M - Q_l(D)Z_l(D)$$

  for some $\bar{A}(D) = [I_M, DI_M, \ldots, D^{\nu_0-1}I_M]^T$. With this change, the new version of (9.86) is

$$I_M - \Theta_1^{*T}\bar{F}(D) - \Theta_2^{*T}F(D)G_0(D) = \Theta_3^* W_m^{-1}(D)G_0(D)$$

  for $\bar{F}(D) = \frac{\bar{A}(D)}{\Lambda(D)}$. It then follows that

$$\lim_{D\to\infty} \Theta_1^{*T}\bar{F}(D) = 0$$

  as $\lim_{D\to\infty} \Theta_2^{*T}F(D)G_0(D) = 0$ and $\lim_{D\to\infty} \Theta_3^* W_m^{-1}(D)G_0(D) = I_M$, so that $\nu_0 - 1 < v$, that is, $\nu_0 = \nu$ is the choice, making $\bar{A}(D) = A(D)$.

  In fact, from (9.83), we have

$$\Lambda(D)K_p^{-1}\xi_m(D)P_l^{-1}(D) = Q_l(D) + R_l(D)P_l^{-1}(D)$$

$$\Lambda(D)K_p^{-1}\xi_m(D)P_l^{-1}(D)Z_l(D) = Q_l(D)Z_l(D) + R_l(D)P_l^{-1}(D)Z_l(D)$$

$$K_p^{-1}\xi_m(D)P_l^{-1}(D)Z_l(D) = \Lambda^{-1}(D)Q_l(D)Z_l(D) + \Lambda^{-1}(D)R_l(D)P_l^{-1}(D)Z_l(D)$$

  so that

$$\begin{aligned} \Theta_1^{*T}\bar{F}(D) &= I_M - \Lambda^{-1}(D)Q_l(D)Z_l(D) \\ &= I_M - K_p^{-1}\xi_m(D)P_l^{-1}(D)Z_l(D) + \Lambda^{-1}(D)R_l(D)P_l^{-1}(D)Z_l(D), \end{aligned}$$

  implying that $\lim_{D\to\infty} \Theta_1^{*T}\bar{F}(D) = 0$.

- On page 393, above (9.106), "(9.102)" should be "(9.105)"; in (9.106), "I" should be "I_M".

- On page 395, after (9.113), "if all finite poles of ..." should be "as all finite poles of ...".

- On page 402, in line 2, "$\Delta_m(s)$" should be "$G_0(s)\Delta_m(s)$".

- On page 406, for (9.161), use $(D-\sigma)H_1(D)[\omega_1](t) = \omega_1(t) - K_1(D)[\omega_1](t)$, $\omega_1(t) = F(D)[u](t)$ and $u(t) = G_0^{-1}(D)[y](t) - \mu\left(\Delta_m(D) + G_0^{-1}(D)\Delta_a(D)\right)[u](t)$.

- On page 408, for (9.173C), use $DH_2(D)[y](t) = y(t) - K_2(D)[y](t)$, (9.172C), (9.168C) and (9.86). For (9.176C), notice that $K_2(D)h^{-1}(D)$ is stable and proper.

  Substituting $u(t)$ of (9.182C) in (9.179C), we have

$$\omega(t) = G_8(D,\cdot)[\bar{y}](t) + G_9(D,\cdot)[r](t)$$

  for some stable and proper operators $G_8(D,t)$ and $G_9(D,t)$, which is used to derive (9.184C) in which the unmodeled dynamics term $\mu(G_0(D)\Delta_m(D) + \Delta_a(D))$ is included in some operators (under the condition that $\mu > 0$ is sufficiently small to make those related operators stable), while not explicitly in (9.184C).

  In (9.180C), "$\beta(\zeta^T\zeta + \xi^T\xi)$" should be "$\beta\varepsilon(\zeta^T\zeta + \xi^T\xi)$".

- On page 410, after (9.171D), add

$$\omega_2(t) = F(D)[y](t) = F(D)(D+a_0)\xi_m^{-1}(D)\frac{\xi_m(D)}{D+a_0}[y](t) = F(D)(D+a_0)\xi_m^{-1}(D)[\hat{y}](t).$$

- On page 410, in (9.174D), "$\beta(\zeta^T\zeta + \xi^T\xi)$" should be "$\beta\varepsilon(\zeta^T\zeta + \xi^T\xi)$", and the second "$\frac{1}{D+a_0}$" should be "$\frac{f(D)}{D+a_0}$". Above (9.174D), change "From (9.129)" to "From (9.129) with $h(D) = \frac{1}{f(D)}$".

- On pages 410-411, to derive (9.184D), the following part is some helpful information.

  From (9.120), we have

$$\begin{aligned}
&\Theta_3^*\xi_m(D)[y-y_m](t)\\
&= (\Theta(t) - \Theta^*)^T\omega(t) + \mu(I - \Theta_1^{*T}F(D))\left(\Delta_m(D) + G_0^{-1}(D)\Delta_a(D)\right)[u](t)\\
&= (\Theta(t) - \Theta^*)^T\omega(t) + \mu(I - \Theta_1^{*T}F(D))\left(\Delta_m(D) + G_0^{-1}(D)\Delta_a(D)\right)D^{-2d_m}[D^{2d_m}[u]](t),
\end{aligned}$$

  where $\omega(t) = [\omega_1^T(t), \omega_2^T(t), r^T(t)]^T$.

  In a similar way, (9.168D) and (9.169D) can be further developed from the current d_m-step form to up to $2d_m$-step forms of the same form in terms of the signals $\omega_1(t)$ and $\omega_2(t)$.

  Then, the above equation can be written as

$$\begin{aligned}
&D^{d_m}\frac{\xi_m(D)}{D+a_0}[y](t) = D^{d_m}[\hat{y}](t)\\
&= D^{d_m}\frac{\xi_m(D)}{D+a_0}[y_m](t) + \Theta_3^{*-1}\frac{D^{d_m}}{D+a_0}[(\Theta - \Theta^*)^T\omega](t)\\
&\quad +\mu\Theta_3^{*-1}\frac{D^{d_m}}{D+a_0}(I - \Theta_1^{*T}F(D))\left(\Delta_m(D) + G_0^{-1}(D)\Delta_a(D)\right)D^{-2d_m}[D^{2d_m}[u]](t)\\
&= G_2(D,t)[\omega_1](t) + G_3(D,t)[\omega_2](t) + d_1(t)\\
&= G_4(D,t)[\hat{y}](t) + d_2(t)
\end{aligned}$$

  for some stable and proper operators $G_i(D,t)$, $i = 2,3,4$, and bounded signals $d_j(t)$, $j = 1,2$.

  The corrected (9.174D), with $h(D) = \frac{1}{f(D)}$ (for simplicity, $f(D) = D^{d_m}$), is

$$\hat{y}(t) = \frac{1}{D+a_0}[r](t) + \frac{f(D)}{D+a_0}[\varepsilon - \Psi\xi + \varepsilon\beta(\zeta^T\zeta + \xi^T\xi) + \alpha\varepsilon m^2](t),$$

where

$$
\begin{aligned}
& f(D)[\varepsilon - \Psi\xi + \varepsilon\beta(\zeta^T\zeta + \xi^T\xi) + \alpha\varepsilon m^2](t) \\
= & \,\varepsilon(t+d_m) - \Psi(t+d_m)\xi(t+d_m) + \varepsilon(t+d_m)\beta(\zeta^T(t+d_m)\zeta(t+d_m) + \xi^T(t+d_m)\xi(t+d_m)) \\
& +\alpha\varepsilon(t+d_m)m^2(t+d_m).
\end{aligned}
$$

With the above analysis, from (9.174D), we can derive (9.184D).

- On page 411, above (9.184C), change "(9.182C)" to "(9.180C), (9.182C)".

  In line 4 below (9.184D), change "such that $\bar{y}(t)$ or $\hat{y}(t)$ is bounded" to "such that $\bar{y}(t)$ is bounded". In the footnote, "(5.332)" should be "(5.331)".

- On page 412, line 2, change "such that $\bar{y}(t)$ or $\hat{y}(t)$ is bounded" to "such that $\hat{y}(t)$ is bounded".

- On page 443, under (9.357), "U" should be "U_s".

- On page 447, in (9.384), "$\tilde{\theta}^T$" should be "$\tilde{\theta}_i^T$".

- On page 448, in (9.389), "$\Phi_i = U\Theta_i$" should be "$\Phi_i^{*T} = U\Theta_i^{*T}$".

- On page 451, in line 1, "(9.397)" should be "(9.395)".

- On page 454, (9.426) should be

$$
\bar{e}(t) = SD_s h(D)\left[\left[\tilde{\phi}_1^T\chi_1, \tilde{\phi}_2^T\chi_2, \ldots, \tilde{\phi}_M^T\chi_M\right]^T\right](t).
$$

- On page 455, under (9.433), "$i = 2,3,\ldots,M$" should be "$i = 1,2,\ldots,M$".

- On page 463, in the second line after (9.486), "${}_mB_m = (S_mB_m)^T > 0$" should be "$S_mB_m = (S_mB_m)^T > 0$".

- One page 479, 4 lines below (9.594), "$\bar{H}$" should "H".

- One page 483, (9.606) should be

$$
u(t) = Y(q, q_d, \dot{q}, \dot{q}_d, \ddot{q}_d, t)\theta^*(t) + \frac{1}{2}\frac{\partial D(q,t)}{\partial t}(\dot{q} + v) - K_D s.
$$

  Note that this control law leads the time derivative of $V(s,\tilde{\theta},t) = \frac{1}{2}s^T D(q(t),t)s$ to $\dot{V} = -s^T K_D s$.

- On page 483, (9.608) should be

$$
m(t) = k_0\|\dot{q}(t) + v(t)\| f(q),\ k_0 > 0,\ \psi(t) = m(t)s(t).
$$

- On page 490, in (9.638), "u_d" should be "u", and in (9.640), "$A(D)Z(D)$" should be "$Z(D)$".

- On page 491, in (9.643), "$A(D)\hat{Z}(D)$" should be "$\hat{Z}(D)$"; at the end of the paragraph before (9.641), "(10.103)–(10.114))" should be "(10.123)–(10.134))".

- On page 492, line 11, "$Q_m(s)[y_m](t) =$ for some" should be "$Q_m(s)[y_m](t) = 0$ for some".

- On page 493, the last line, "K_3^*" should be "Θ_3^*".

- On page 494, below (9.655), "$\varepsilon(t) = \tilde{\Theta}^T(t)\zeta](t)$" should be "$\varepsilon(t) = \tilde{\Theta}^T(t)\zeta(t)$"; also in (9.656), "$\tilde{\Theta}^T(t)\zeta]$" should be "$\tilde{\Theta}^T(t)\zeta(t)$".

  To see (9.655), we derive

$$\tilde{\Theta}^T(t)f(D)[\zeta](t) = \tilde{\Theta}^T(t)\omega(t) + \tilde{\Theta}_3(t)(\xi_m(D)[y](t) - r(t)) = \Theta_3(t)\xi_m(D)[y - y_m](t)$$

  here the identity $\tilde{\Theta}^T(t)\omega(t) = \Theta_3^*\xi_m(D)[y - y_m](t)$ has been used.

- On page 494, above (9.657), "(9.468)" should be "(9.366)".

- On page 495, in and below (9.661), "$\Phi(t)$" should be "$\Phi(t-1)$".

- On pages 499-500, Problem 9.34, for part (iii), replace the adaptive law by: $\tilde{\Theta}^T(t+1) = \tilde{\Theta}^T(t) - \frac{\Gamma\varepsilon(t)\zeta^T(t)}{1+\zeta^T(t)\zeta(t)}$; for part (iv), replace the adaptive law by: $\tilde{\Theta}^T(t+1) = \tilde{\Theta}^T(t) - \frac{\varepsilon(t)\zeta^T(t)}{1+\zeta^T(t)\zeta(t)}$, note that the condition on Γ is: $\Gamma K_p + K_p^T\Gamma^T - K_p^T\Gamma K_p > 0$, as

$$\begin{aligned}
&\mathrm{tr}[\tilde{\Theta}(t+1)\Gamma\tilde{\Theta}^T(t+1)] - \mathrm{tr}[\tilde{\Theta}(t)\Gamma\tilde{\Theta}^T(t)] \\
&= \frac{1}{1+\zeta^T(t)\zeta(t)}\varepsilon^T(t)\left(-\Gamma K_p^{-1} - (\Gamma K_p^{-1})^T + \frac{\zeta^T(t)\zeta(t)}{1+\zeta^T(t)\zeta(t)}\Gamma\right)\varepsilon(t) \\
&= -\frac{1}{1+\zeta^T(t)\zeta(t)}\varepsilon^T(t)(K_p^T)^{-1}\left(K_p^T\Gamma + \Gamma K_p - \frac{\zeta^T(t)\zeta(t)}{1+\zeta^T(t)\zeta(t)}K_p^T\Gamma K_p\right)K_p^{-1}\varepsilon(t) \le 0;
\end{aligned}$$

  for part (v), replaced it by: "(v) Repeat parts (i) and (ii) with a normalized adaptive law, for example, $\dot{\tilde{\Theta}}^T(t) = -\frac{\Gamma\varepsilon(t)\zeta^T(t)}{1+\zeta^T(t)\zeta(t)}$, and parts (iii) and (iv) with an unnormalized adaptive law, for example, $\tilde{\Theta}^T(t+1) = \tilde{\Theta}^T(t) - \Gamma\varepsilon(t)\zeta^T(t)$, and comment on the results."

- On page 526, after (10.109), the sentence "where $u_d(t) = [u_{d1}(t), \ldots, u_{dm}(t)]^T$ is the vector input signal to be generated from a desired control law for $\widehat{NI}(\cdot) = [\widehat{NI}_1(\cdot), \ldots, \widehat{NI}_m(\cdot)]^T$" is replaced by "where $u_d(t) = [u_{d1}(t), \ldots, u_{dM}(t)]^T$ is generated from a desired control law for $\widehat{NI}(\cdot) = [\widehat{NI}_1(\cdot), \ldots, \widehat{NI}_M(\cdot)]^T$".

- On page 528, in (10.134), "$\sum_{i=1}^m$" should be "$\sum_{i=1}^M$".

- On page 529, in (10.140) and (10.141), "$\omega_1(t)$" should be "$\psi_1(t)$", and "$\omega_2(t)$" should be "$\psi_2(t)$".

- On page 530, after (10.148), ϕ_4^* should be "$\phi_4^* = -\phi_1^* \otimes \theta^* \stackrel{\triangle}{=} -[\phi_{11}^*\theta^{*T}, \phi_{12}^*\theta^{*T}, \ldots, \phi_{1n-1}^*\theta^{*T}]^T \in R^{(n-1)n_\theta}$".

- On page 531, after (10.155), "and $d(t)$ is as in (10.77)" should be "and $d(t)$ is as in (10.77) with $W(D)$ in (10.87)".

- On page 533, in (10.170), "$d(t)$" should be "$d_N(t)$".

- On page 534, in line 5, "x" should be "x_m"; in (10.171), "s" should be "D"; above (10.173), "$\omega(t)$" should be "$\omega_N(t)$"; in (10.173), "ω" should be "ω_N and "ψ_4" should be "ω_4"; in (10.175), "ψ_4" should be "ω_4"; in (10.176), "Φ^3" should be "Φ_3"; and in (10.177), "ψ_4" should be "ω_4" and "ψ_2" should be "ω_2" and .

- On page 538, below (10.192), add "making $\alpha(x)$ in (10.191) nonsingular" after "where r_i is such that $L_{g_j}L_f^{r_i}h_i(x) \neq 0$, $1 \leq j \leq M$".

- On page 556, (10.288) should be

$$\omega_{m2}(t) = \frac{(s^{m+1} + k_1 s^m)I_4}{s^n + k_1 s^{n-1} + \cdots + k_{n-1}s + k_n}[\sigma(y)\omega](t) \text{ (for } \rho \geq 2).$$

  (Note that for the case of $n - m = 1$, $\omega_{m2}(t) = \frac{-(k_2 s^{n-2} + k_3 s^{n-3} + \cdots + k_{n-1}s + k_n)I_4}{s^n + k_1 s^{n-1} + \cdots + k_{n-1}s + k_n}[\sigma(y)\omega](t)$, and no derivative of $\omega_{i2}(t)$, $i = 0, 1, \ldots, m$, is needed in the backstepping design procedure.)

- On page 569, at the end, add "Since $r_i > 0$ and $s_{ki} > 0$ in (10.328) are design parameters, they can be chosen to be larger to make the effect of $\frac{d^2}{4r_1} + \sum_{i=2}^{\rho} \frac{d^2}{4r_i}$ and $\sum_{k=2}^{\rho-1}\sum_{i=1}^{n_\theta} \frac{\chi_i^2}{4s_{ki}}$ on $\dot{V}_\rho$ smaller, to reduce the tracking error $z_1(t) = y(t) - y_m(t)$."

- On page 596, in [233], "with" should be "without".