ECE 6852: Linear State Space Control Systems
MAE 6620: Linear State Space Systems
SYS 6012: Dynamic Systems (Fall 2022, 3 credits)

Objective:

Study the state variable approach for modeling, analysis and control of linear dynamic systems

Description:

State variables and system matrices are used to describe system dynamics and behaviors. State-space solutions are obtained to represent system responses to initial conditions and external inputs. System stability, controllability, observability are investigated using system matrices to characterize key system properties. State feedback control laws are designed to achieve desired system performance. State observers are employed to estimate unmeasured system state variables for feedback control. Physical system examples are used to illustrate various concepts of the state variable approach. Matlab simulations are used to help the control system design and analysis.

Prerequisites:

ECE4851 or ECE6851 or equivalent.

Instructor:

Professor Gang Tao, ECE Department, Thornton Hall E311, 924-4586, gt9s@virginia.edu.

Lecture hours:

2:00 - 3:15 pm, Mondays and Wednesdays (in room MEC 305).

Office hours:

11:00 - 12:15, Monday; 2:00 - 3:15, Tuesday.

Textbook:

Chi-Tsong Chen, *Linear System Theory and Design*, 4th edition, Oxford University Press, 2013 (its 3rd edition (1999) can also be used) (required).

Topics:

1. System modeling (2 lectures)
2. Matrix theory (2 lectures)
3. State transition matrix (2 lectures)
4. State-space solutions (2 lectures)
5. Stability (2 lectures)
6. Controllability (2 lectures)
7. Observability (2 lectures)
8. Canonical forms (2 lectures)
9. System realization (2 lectures)
10. State feedback control (3 lectures)
11. State observer (2 lectures)
12. Output feedback control (3 lecture).

References:

[1] Thomas Kailath, *Linear Systems*, Prentice Hall, 1980
[2] Wilson J. Rugh, *Linear System Theory*, 2nd ed., Prentice Hall, 1996.

Course work:

Homeworks: 20 %; Midterm exam: 30 %; Project: 10 %; Final exam: 40 %.

Additional information:

Disability accommodations

UVA is committed to creating a learning environment that meets the needs of its diverse student body. If you anticipate or experience any barriers to learning in this course, please feel welcome to discuss your concerns with me. If you have a disability, or think you may have a disability, you may also want to meet with the Student Disability Access Center (SDAC), to request an official accommodation. You can find more information about SDAC, including how to apply online, through their website at

`http://sdac.studenthealth.virginia.edu`

If you have already been approved for accommodations through SDAC, please make sure to send me your accommodation letter and meet with me so we can develop an implementation plan together.

Violence and sexual assault prevention

The University of Virginia is dedicated to providing a safe and equitable learning environment for all students. For information about violence prevention and sexual assault prevention, please see

`https://notonourgrounds.virginia.edu`
`https://notonourgrounds.virginia.edu/greendot`
`https://uvapolice.virginia.edu/sexual-assault`
`https://eocr.virginia.edu`

Religious accommodations

It is the University's long-standing policy and practice to reasonably accommodate students so that they do not experience an adverse academic consequence when sincerely held religious beliefs or observances conflict with academic requirements.

Students who wish to request academic accommodation for a religious observance should submit their request to me by email as far in advance as possible. If you have questions or concerns about your request, you can contact the University's Office for Equal Opportunity and Civil Rights (EOCR) at UVAEOCR@virginia.edu or 434-924-3200. Accommodations do not relieve you of the responsibility for completion of any part of the coursework you miss as the result of a religious observance.

For information about accommodations for religious observance, please see

`https://eocr.virginia.edu/accommodations-religious-observance`

UVa honor system

For information about University of Virginia's honor code, please see

`http://honor.virginia.edu`
`https://honor.virginia.edu/frequently-asked-questions`

In this class, we follow the honor statement suggested in

`https://honor.virginia.edu/statement`

In particular, all graded assignments (homeworks, project and tests) should be pledged, and for any homework, project or test, no old or new solution can be consulted before your own solution is turned in. Group discussion is allowed for homeworks only, not for the project and tests.

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Homework Assignments, and Project and Test Schedule

Honor code and solution policy:

All graded assignments (homework, project and test) should be pledged, and for any homework, project or test, no solution can be consulted before your own solution is handed in. Group discussion is allowed for homeworks only, not for the project and tests for which additional guidelines will be given.

Late work policy:

Unless an extension has been given ahead of time for special situations, no late project or test will be accepted, and a late homework submission will be subject to penalty: 10% if the submission is up to 12 hours late, 25% if the submission is up to the next 12 hours late, and 100% if the submission is more than 24 hours late. Any grading dispute should be discussed with the instructor and/or the TA as soon as possible and no later than one week, to avoid any potential delay in the next grading and final grade.

Homework assignments:

The problem numbers are referred to the problem numbers in each chapter (the numbers in the parentheses are for the 3rd edition of the textbook, e.g., 2.8 (2.20) means Problem 2.8 of the 4th edition or Problem 2.20 of the 3rd edition). For some problems, solution hints are provided (in the parentheses after the problem number) to help the understanding of the tasks of the homework.

Each homework should be submitted in a .pdf document, via email, to Mr. Qianhong Zhao (our teaching assistant), at qz2nv@virginia.edu, by 4:30pm on one selected Wednesday (see next).

Homework 1: Problem 2.5 (2.5) (hint: the system under consideration is a linear system), 2.8 (2.20) (hint: to derive its transfer function (matrix), assume all circuit parameter values are equal to 1 for simplicity), 2.14 (2.10), and 2.18 (2.15) (only the system in Figure (b)) (hint: use the system differential equations: $u\cos\theta_2 - m_2 g\sin\theta_2 = m_2 l_2 \ddot{\theta}_2$ and $T\sin(\theta_2-\theta_1) - m_1 g\sin\theta_1 = m_1 l_1 \ddot{\theta}_1$ with $T = m_2 g\cos\theta_2 + u\sin\theta_2$ (where g is the gravitational acceleration), the state variables: $x_1 = \theta_1$, $x_2 = \dot{\theta}_1$, $x_3 = \theta_2$, $x_4 = \dot{\theta}_2$, and the interpretation that θ_1 and θ_2 being small means that $\sin\theta_i \approx \theta_i$ and $\theta_j\theta_k \approx 0$, $i, j, k = 1, 2$)

Due by **4:30pm, September 7, Wednesday**.

Homework 2: Problem 3.5 (3.5) (do not use Matlab), 3.7 (3.7), 3.8 (3.8) (revision: for each part of (3.5), (3.7) and (3.8) of the 3rd edition textbook, multiply the third column vector by -1, to match that of the

4th edition), 3.17 (3.17) (<u>correction</u>: the (1,2) element of Q in Problem 3.17 should be: $\frac{\lambda T^2}{2}$), 3.21 (3.21), 3.33 (3.33).

Due by **4:30pm, September 14, Wednesday**.

Homework 3: Problem 4.1 (4.1), 4.16 (4.16) (<u>correction</u>: for the solution given on page 375 (325), the (1,2) element of $\Phi(t,t_0)$ of 4.16(b) should be: $0.5(e^{t+t_0}-e^{3t_0-t})$, not $0.5(e^{t+t_0}-e^{t_0-t})$), 4.18 (4.18) (<u>hint</u>: use the definition of $\Phi(t,t_0)$, express $\frac{d}{dt}(\det(\Phi(t,t_0)))$ in terms of $\det(\Phi(t,t_0))$ and $a_{ij}(t)$, and solve the resulted equation for $\det(\Phi(t,t_0))$; or use the result for the Wronskian $W(t)=\det(X(t))$ (see Lecture notes for fundamental solutions)), 4.20 (4.20), and the following additional problems (you may only use Matlab to verify your result):

(1) Using our three methods, find the state transition matrix $\Phi(t,t_0)$ of $\dot{x}=Ax$ with $A=\begin{bmatrix} 2.5 & 0.5 \\ -0.5 & 1.5 \end{bmatrix}$.

(2) For the system $\dot{x}=Ax+Bu$, $y=Cx$ with the above A, $B=\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $C=\begin{bmatrix} 2 & 5 \end{bmatrix}$, express $x(t)$ and $y(t)$ in terms of $u(t)$ and $x(0)=x_0$ (that is, the state-space solution).

(3) Find the system impulse response functions and the system transfer functions, for x and y.

(4) For $\bar{x}=Px$ with $P=\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the system dynamic equation in terms of $\bar{x}$.

(5) For the system in (4) above, express $\bar{x}(t)$ and $y(t)$ in terms of $u(t)$ and $x(0)=x_0$.

Due by **4:30pm, September 28, Wednesday**.

Homework 4: Problem 5.9 (5.7: with $A=\begin{bmatrix} -1 & 5 \\ 0 & 2 \end{bmatrix}$, $B=\begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $C=\begin{bmatrix} -2 & 4 \end{bmatrix}$ and $D=-2$), 5.11 (5.9) (the concept of "being marginally stable" is what we studied as the concept of "being stable" or "being Lyapunov stable" for the internal stability, as in the literature), 5.12 (5.10: $a_{13}=2$), 5.13 (5.11: $a_{13}=2$), and the following additional problems:

(1) Check the output natural response stability of

$$y^{(4)}(t)+2y^{(2)}(t)+y(t)=0.$$

(2) Check the Lyapunov stability of $\dot{x}=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & -2 & 0 \end{bmatrix}x$.

(3) Check the Lyapunov stability of $\dot{x}=\begin{bmatrix} 1 & -2 & 2 & -2 \\ 1 & -1 & 1 & -2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & -1 \end{bmatrix}x$.

(4) For the system transfer matrix $G(s)=\begin{bmatrix} \frac{1}{s+1} & \frac{1}{s+2} \\ \frac{1}{s+3} & \frac{1}{s+4} \end{bmatrix}$, find its L^1 and L^∞ operator norms.

(5) For the system transfer function $g(s) = \frac{s+2}{(s+1)(s+4)}$, find its L^2 operator norm (H^∞ norm).

Due by **4:30pm, October 5, Wednesday**.

Reading days: October 1 - 4 (Saturday - Tuesday)

Test 1 (Topics 1-5, Homeworks1-4): **October 12, Wednesday, 6:00-8:00pm** (a closed-book test for which only two pages of reference notes allowed).

Homework 5: Problem 6.1 (6.1: $A = \begin{bmatrix} -2 & 3 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 3 \end{bmatrix}$), 6.2 (6.2: with $b_{12} = 2$, $c_{13} = -2$), 6.6 (6.6), 6.15 (6.15) (hint: use Theorem 6.8), and the following additional problems:

(1) For the pair (A, B), with $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -4 & -6 & -4 \end{bmatrix}^T$ and $B = B_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$, derive its controllability decomposition, and repeat it for $B = B_2 = [1, 1, 0, 0]^T$.

(2) For ($A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -4 & -6 & -4 \end{bmatrix}, C = \begin{bmatrix} 1 & 3 & 3 & 1 \end{bmatrix}$), derive its observability decomposition.

For (1) and (2), you may use Matlab. Please specify the submatrices in $\bar{A}$, $\bar{B}$ and $\bar{C}$ in each decomposition to show the controllable/uncontrollable or observable/unobservable parts of the system and the corresponding system modes (eigenvalues).

Due by **4:30pm, October 19, Wednesday**.

Homework 6: (1) Consider the system $\dot{x} = Ax + Bu$, $y = Cx$, with

$$\begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix}.$$

(i) Find its transfer function;

(ii) find its controller canonical form (what is called in the textbook as "the controllable canonical form") and the associated state transformation matrix P;

(iii) find its controllable canonical form (we studied; what is commonly called in the literature) and the associated state transformation matrix P;

(iv) find its observer canonical form (what is called in the textbook as "the observable canonical form") and the associated state transformation matrix P; and

(v) find its observable canonical form (we studied; what is commonly called in the literature) and the associated state transformation matrix P.

(2) Consider the system $\dot{x} = Ax + Bu$, with

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A = [1    -2     1     0     1;
     1     0     2     1    -1;
     1     0     0     1    -1;
     1     1     1    -1     0;
     0     2     1    -1     0]
B = [1     1;
     2    -1;
     1     0;
     0     1;
     1    -1]
```

Find its controllable canonical form (A_c, B_c) and the associated transformation matrix P.

Due by **4:30pm, October 26, Wednesday**.

Homework 7: (1) Consider the transfer function $G(s) = \frac{s+2}{s^3+2s^2-s-2}$.

(i) Find the controller canonical form realization (A_c, B_c, C_c) of the given $G(s)$, check the observability of (A_c, C_c), and perform the observability decomposition on (A_c, B_c, C_c).

(ii) Find a minimal realization of $G(s)$, from the observability decomposition on (A_c, B_c, C_c).

(iii) Find a minimal realization of $G(s)$, by direct use of the coprime fraction of $G(s)$.

(iv) Find an equivalence transformation which connects the above two minimal realizations of $G(s)$.

Hints: see lectures; in particular, for (iii), use Theorem 7.2, and for (iv) use Theorem 7.3.

(2) 7.16 (7.13).

(3) For $G_1(s)$ in 7.16 (7.13), find its controller canonical form realization, and perform an observability decomposition on the controller form realization, to obtain a minimal realization of $G_1(s)$.

Due by **4:30pm, November 2, Wednesday**.

Homework 8: 8.1 (8.1), 8.2 (8.2), 8.4 (8.4), 8.7 (8.7), 8.13 (8.13) (replace the eigenvalues $-5 \pm 4j$ of $A - BK$, by -5 and -6; use Matlab; use the designs in the lecture notes; verify it).

Due by **4:30pm, November 9, Wednesday**.

Project: **November 16 - November 28** (Wednesday - Monday) (a take-home and open-book project).

Test 2: **December 5 - December 12** (Monday - Monday) (a take-home and open-book test).