School of Engineering and Applied Science
Department of Electrical and Computer Engineering

ECE6851/MAE6610 – Linear Automatic Control Systems (Fall 2022, 3 credits)

Objective:

Study the theory and techniques for design and analysis of linear feedback control systems.

Description:

This course is to explore modeling of linear dynamic systems (using input-output representations and state space representations described by differential equations and transfer functions), to analyze control systems in both time and frequency domains, to study properties of feedback control systems, to investigate system stability using the Routh-Hurwitz criterion, Nyquist criterion, root-locus and Bode plots, and to design PID, lead, lag and state feedback controllers to improve system performance.

Prerequisites:

ECE 3750 (Signals and Systems) or equivalent.

Instructor:

Prof. Gang Tao, Thornton Hall, Room E311, (434) 924-4586, gt9s@virginia.edu.

Lecture hours:

11:00 - 12:15, Tuesdays and Thursdays (classroom: THN D115)

Textbook:

G. F. Franklin, J. D. Powell and A. Emami-Naeini, *Feedback Control of Dynamic Systems*, 7th ed., Pearson Prentice Hall, Upper Saddle River, NJ, 2014 (or the 6th ed. (2009), or the 8th ed. (2019)) (required).

Topics:

1. Mathematical models of control systems (2 lectures)
2. Transfer functions and block diagrams (2 lectures)
3. Time-domain responses (2 lectures)
4. The Routh-Hurwitz stability criterion (2 lectures)
5. Feedback control systems (2 lectures)
6. Root-locus techniques (5 lectures)
7. Bode plot techniques (2 lectures)
8. The Nyquist stability criterion (2 lectures)
9. Dynamic compensation in frequency-domain (3 lectures)
10. State space analysis and pole placement design (4 lectures).

References:

1. R. C. Dorf, *Modern Control Systems*, Prentice-Hall, 12th ed., 2011.
2. N. S. Nise, *Control Systems Engineering*, Wiley, 6th ed., 2011.

Grading:

1. Homeworks: 20 %; 2. Test 1: 30 %; 3. Project: 20%; 4. Test 2: 30 %.

Additional information:

Disability accommodations

UVA is committed to creating a learning environment that meets the needs of its diverse student body. If you anticipate or experience any barriers to learning in this course, please feel welcome to discuss your concerns with me. If you have a disability, or think you may have a disability, you may also want to meet with the Student Disability Access Center (SDAC), to request an official accommodation. You can find more information about SDAC, including how to apply online, through their website at

`http://sdac.studenthealth.virginia.edu`

If you have already been approved for accommodations through SDAC, please make sure to send me your accommodation letter and meet with me so we can develop an implementation plan together.

Violence and sexual assault prevention

The University of Virginia is dedicated to providing a safe and equitable learning environment for all students. For information about violence prevention and sexual assault prevention, please see

`https://notonourgrounds.virginia.edu`
`https://notonourgrounds.virginia.edu/greendot`
`https://uvapolice.virginia.edu/sexual-assault`
`https://eocr.virginia.edu`

Religious accommodations

It is the University's long-standing policy and practice to reasonably accommodate students so that they do not experience an adverse academic consequence when sincerely held religious beliefs or observances conflict with academic requirements.

Students who wish to request academic accommodation for a religious observance should submit their request to me by email as far in advance as possible. If you have questions or concerns about your request, you can contact the University's Office for Equal Opportunity and Civil Rights (EOCR) at uvaeocr@virginia.edu or 434-924-3200. Accommodations do not relieve you of the responsibility for completion of any part of the coursework you miss as the result of a religious observance.

For information about accommodations for religious observance, please see

`https://eocr.virginia.edu/accommodations-religious-observance`

UVa honor system

For information about University of Virginia's honor code, please see

`http://honor.virginia.edu`
`https://honor.virginia.edu/frequently-asked-questions`

In this class, we follow the honor statement suggested in

`https://honor.virginia.edu/statement`

In particular, all graded assignments (homeworks, project and tests) should be pledged, and for any homework, project or test, no old or new solution can be consulted before your own solution is turned in. Group discussion is allowed for homeworks only, not for the project and tests.

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Schedule of Office Hours, Homeworks, Project and Tests

Instructor's office hours: 11:00 - 12:15, Wednesday; 2:00 - 3:15, Thursday.

Honor code and solution policy:

All graded assignments (homework, project and test) should be pledged, and for any homework, project or test, no solution can be consulted before your own solution is handed in. Group discussion is allowed for homeworks only, not for the project and tests for which additional guidelines will be given.

Late work policy:

Unless an extension has been given ahead of time for special situations, no late project or test will be accepted, and a late homework submission will be subject to penalty: 10% if the submission is up to 12 hours late, 25% if the submission is up to the next 12 hours late, and 100% if the submission is more than 24 hours late. Any grading dispute should be discussed with the instructor and/or the TA as soon as possible and no later than one week, to avoid any potential delay in the next grading and final grade.

Grading policy: Homeworks: 20 %; Test 1: 30 %; Project: 20%; Test 2: 30 %.

Each homework should be submitted in a .pdf document, via email, to Mr. Qianhong Zhao (our teaching assistant), at qz2nv@virginia.edu, by 4:00pm on one selected Thursday (most times) or Tuesday.

Homework 1: Due by **4:00pm, September 1, Thursday**.

Homework 2: Due by **4:00pm, September 8, Thursday**.

Homework 3: Due by **4:00pm, September 15, Thursday**.

Homework 4: Due by **4:00pm, September 22, Thursday**.

Homework 5: Due by **4:00pm, September 29, Thursday**.

Reading days: October 1 - 4 (Saturday - Tuesday)

Test 1 (Topics 1 - 5): **October 13, Thursday, 6:00 - 8:00pm** (a closed-book test for which only two pages of reference notes are allowed).

Homework 6: Due by **4:00pm, October 20, Thursday**.

Homework 7: Due by **4:00pm, October 27, Thursday**.

Project: **October 27 - November 3** (Thursday - Thursday) (a take-home and open-book project).

Homework 8: Due by **4:00pm, November 10, Thursday**.

Homework 9: Due by **4:00pm, November 22, Tuesday**.

Homework 10: Due by **4:00pm, December 6, Tuesday**.

Test 2 (Topics 7 - 10): **December 9, Friday, 10:00am - 12:00pm** (an open-book test).

ECE6851/MAE6610 – **Linear Automatic Control Systems** (Fall 2022; 3 credits)

Homework Assignments Based on the 6th Edition Textbook

Homework assignments ("1.1" or "2.9(a)" = "Problem 1.1 or 2.9 part (a) of the 6th edition textbook):

Homework 1 (Topic 1): 1.1, 2.9(a), 2.20, 9.1. (Hint for 9.1: with $x_1 = \theta$, $x_2 = \dot{\theta}$, do linearization at $\theta = \dot{\theta} = 0$.)

Reading assignment: Section 9.2.1 – Linearization

Homework 2 (Topic 2): 3.3(c), 3.7(d) ($F(s) = \frac{3s^2+9s+12}{(s+1)(s^2+5s+11)}$), (i), 2.9(b), 3.15 (the input is v_a), 3.20.

Homework 3 (Topic 3): 3.25, 3.28 (hint: using the region $\omega_1 \leq \omega_n \leq \omega_2$, $\theta_1 \leq \theta \leq \theta_2$ containing the given circle), 3.35, 3.38 (just use the given expression of $y(t)$ to work out parts (a) - (d); no need to derive it).

Homework 4 (Topic 4): 3.42 (hint: $KG(s)$ is the open-loop transfer function), 3.43, 3.45; and, in addition,

Investigate the relative stability concept: (a) design a scheme to test the root locations of an nth-order polynomial equation $a(s) = 0$ relative to the axis $Re[s] = -\delta$ for a given number $\delta > 0$ (that is, to find how many zeros of $a(s)$ are in $Re[s] < -\delta$, on $Re[s] = -\delta$ and in $Re[s] > -\delta$), and (b) illustrate your scheme by a third-order example.

Homework 5 (Topic 5): 4.2, 4.31 (hint: for (c), (e) and (g), only evaluate the steady-state value of $\theta(t)$ for $w(t) = w_0$ and $\theta_r(t) = 0$, and do not determine system type and error constant; for (a), (b), (d), (f), set $w(t) = 0$; for all parts, check system stability first and then answer what the book asks for), 4.19, 4.24(a), (b) ($K = k_p$, $H_f = H_y$), (c) (the feed forward case is the open-loop case).

Homework 6 (Topic 6a): 5.4(c), 5.6(d) (with $L(s) = \frac{s+3}{s^2(s+10)}$), 5.7(a), 5.8(b), 5.41 (for 5.7(a), 5.8(b)).

Homework 7 (Topic 6b): 5.25, 5.26, 5.30 (hint: for (a), consider a positive K for positive feedback and use root locus technique in Matlab).

Homework 8 (Topics 7 and 8): 6.3(c), (d) (with $L(s) = \frac{1}{(s+1)^2(s+10)^2}$), (e) (with $L(s) = \frac{10(s+4)}{s(s+1)(s+200)}$), (h) (with $L(s) = \frac{4s(s+10)}{(s+100)(s+500)}$) (hint: obtain the Bode plots by Matlab), 6.17(b), 6.19(a)-(d) (hint: for (a)-(b), also draw Nyquist plot of $G(s)$ by hand using our method and verify it by Matlab; for (d), find the range of K for $1+KG(s)$ to have all zeros stable, using our method).

For 6.3(c) and (e), also figure out the gain margin and phase margin of each case at $K = 1$, for the closed-loop unity negative feedback system with open-loop transfer function $KL(s)$. Also, for each case, examine the closed-loop stability for $K = 1$. You may use the "grid" command on the Bode plots to make it easy to figure out the numbers; note that the amplitude Bode plots from Matlab are given in db: $20\log|L(j\omega)|$, e.g., $20\log(1) = 0$, and you need to figure out the original $|L(j\omega)|$.

Homework 9 (Topic 9): 6.49, 6.50, 6.60 (hint: use $VM = \min_\omega 1/|S(j\omega)| = 1/\max_\omega |S(j\omega)|$).

Reading assignment: Sections 6.7.7 and 6.7.8 – Sensitivity

Additional problems: (1) Draw the Nyquist plot of $G(s) = \frac{s+5}{(s-2)(s-4)}$ by hands, and find the range of K for the system $\frac{KG(s)}{1+KG(s)}$ to be stable (you may use Matlab to verify your results) (hint: use the procedure in the lecture notes). (2) Repeat the above work for $G(s) = \frac{(s+3)(s+5)}{(s-2)(s-4)}$.

Homework 10 (Topic 10): 7.17(b), 7.20, 7.21 (hint: $y = x_1$), 7.48(a), (b), (c), 9.28 (hint: from the closed-loop transfer function, put the system in the state-space controller canonical form, to get the A matrix first.)

Reading assignment: Section 9.5.2 – Lyapunov stability

ECE6851/MAE6610 – **Linear Automatic Control Systems** (Fall 2022; 3 credits)

Homework Assignments Based on the 7th Edition Textbook

Homework assignments (“1.1” or “2.8(a)” = “Problem 1.1 or 2.8 part (a) of the 7th edition textbook):

Homework 1 (Topic 1): 1.1, 2.8(a), 2.20, 9.1. (Hint for 9.1: with $x_1 = \theta$, $x_2 = \dot{\theta}$, do linearization at $\theta = \dot{\theta} = 0$; for 9.2(b), the equation should be revised as: $\ddot{\theta} + \frac{g}{l}\theta = 0$.)

Reading assignment: Section 9.2.1 – Linearization

Homework 2 (Topic 2): 3.3(c), 3.7(d) ($F(s) = \frac{3s^2+9s+12}{(s+1)(s^2+5s+11)}$), (i), 2.8(b), 3.15 (the input is v_a), 3.20.

Homework 3 (Topic 3): 3.27, 3.31 (hint: using the region $\omega_1 \leq \omega_n \leq \omega_2$, $\theta_1 \leq \theta \leq \theta_2$ containing the given circle), 3.42, 3.49 (just use the given expression of $y(t)$ to work out parts (a) - (d); no need to derive it).

Homework 4 (Topic 4): 3.53 (hint: $KG(s)$ is the open-loop transfer function), 3.54, 3.56; and, in addition, Investigate the relative stability concept: (a) design a scheme to test the root locations of an nth-order polynomial equation $a(s) = 0$ relative to the axis $Re[s] = -\delta$ for a given number $\delta > 0$ (that is, to find how many zeros of $a(s)$ are in $Re[s] < -\delta$, on $Re[s] = -\delta$ and in $Re[s] > -\delta$), and (b) illustrate your scheme by a third-order example.

Homework 5 (Topic 5): 4.2, 4.34 (hint: for (c), (e) and (g), only evaluate the steady-state value of $\theta(t)$ for $w(t) = w_0$ and $\theta_r(t) = 0$, and do not determine system type and error constant; for (a), (b), (d), (f), set $w(t) = 0$; for all parts, check system stability first and then answer what the book asks for), 4.21, 4.27(a), (b) ($K = k_p$, $H_f = H_y$), (c) (the feed forward case is the open-loop case).

Homework 6 (Topic 6a): 5.4(d), 5.6(d) (with $L(s) = \frac{s+3}{s^2(s+10)}$), 5.7(a), 5.8(b), 5.42 (for 5.7(a), 5.8(b)).

Homework 7 (Topic 6b): 5.25, 5.26, 5.30 (hint: for (a), consider a positive K for positive feedback and use root locus technique in Matlab).

Homework 8 (Topics 7 and 8): 6.3(c), (d), (e) (with $L(s) = \frac{10(s+4)}{s(s+1)(s+200)}$), (h) (hint: obtain the Bode plots by Matlab), 6.17(b), 6.19(a)-(d) (hint: for (a)-(b), also draw Nyquist plot of $G(s)$ by hand using our method and verify it by Matlab; for (d), find the range of K for $1 + KG(s)$ to have all zeros stable, using our method).

For 6.3(c) and (e), also figure out the gain margin and phase margin of each case at $K = 1$, for the closed-loop unity negative feedback system with open-loop transfer function $KL(s)$. Also, for each case, examine the closed-loop stability for $K = 1$. You may use the “grid” command on the Bode plots to make it easy to figure out the numbers; note that the amplitude Bode plots from Matlab are given in db: $20\log|L(j\omega)|$, e.g., $20\log(1) = 0$, and you need to figure out the original $|L(j\omega)|$.

Homework 9 (Topic 9): 6.49, 6.50, 6.60 (hint: use $VM = \min_\omega 1/|S(j\omega)| = 1/\max_\omega |S(j\omega)|$).

Reading assignment: Sections 6.7.7 and 6.7.8 – Sensitivity.

Additional problems: (1) Draw the Nyquist plot of $G(s) = \frac{s+5}{(s-2)(s-4)}$ by hands, and find the range of K for the system $\frac{KG(s)}{1+KG(s)}$ to be stable (you may use Matlab to verify your results) (hint: use the procedure in the lecture notes). (2) Repeat the above work for $G(s) = \frac{(s+3)(s+5)}{(s-2)(s-4)}$.

Homework 10 (Topic 10): 7.17(b), 7.21, 7.22 (hint: $y = x_1$), 7.49(a), (b), (c), 9.29 (hint: from the closed-loop transfer function, put the system in the state-space controller canonical form, to get the A matrix first.)

Reading assignment: Section 9.5.2 – Lyapunov stability

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Homework Assignments Based on the 8th Edition Textbook

Homework assignments ("1.1" or "2.8(a)" = "Problem 1.1 or 2.8 part (a) of the 8th edition textbook):

Homework 1 (Topic 1): 1.1, 2.8(a), 2.22, 9.2. (Hint for 9.2: with $x_1 = \theta$, $x_2 = \dot{\theta}$, do linearization at $\theta = \dot{\theta} = 0$; for 9.2(b), the equation should be revised as: $\ddot{\theta} + \frac{g}{l}\theta = 0$.)

Reading assignment: Section 9.2.1 – Linearization

Homework 2 (Topic 2): 3.3(c), 3.7(d), (i), 2.8(b), 3.14 (the input is v_a), 3.19.

Homework 3 (Topic 3): 3.26, 3.30 (hint: using the region $\omega_1 \leq \omega_n \leq \omega_2$, $\theta_1 \leq \theta \leq \theta_2$ containing the given circle), 3.41, 3.48 (just use the given expression of $y(t)$ to work out parts (a) - (d); no need to derive it).

Homework 4 (Topic 4): 3.52 (hint: $KG(s)$ is the open-loop transfer function), 3.53, 3.55; and, in addition, Investigate the relative stability concept: (a) design a scheme to test the root locations of an nth-order polynomial equation $a(s) = 0$ relative to the axis $Re[s] = -\delta$ for a given number $\delta > 0$ (that is, to find how many zeros of $a(s)$ are in $Re[s] < -\delta$, on $Re[s] = -\delta$ and in $Re[s] > -\delta$), and (b) illustrate your scheme by a third-order example.

Homework 5 (Topic 5): 4.2, 4.44 (hint: for (c), (e) and (g), only evaluate the steady-state value of $\theta(t)$ for $w(t) = w_0$ and $\theta_r(t) = 0$, and do not determine system type and error constant; for (a), (b), (d), (f), set $w(t) = 0$; for all parts, check system stability first and then answer what the book asks for), 4.21, 4.27(a), (b) ($K = k_p$, $H_f = H_y$), (c) (the feed forward case is the open-loop case).

Homework 6 (Topic 6a): 5.4(d), 5.6(d), 5.7(a), 5.8(b), 5.42 (for 5.7(a), 5.8(b)).

Homework 7 (Topic 6b): 5.24, 5.25, 5.29 (hint: for (a), consider a positive K for positive feedback and use root locus technique in Matlab).

Homework 8 (Topics 7 and 8): 6.3(c), (d), (e), (h) (hint: obtain the Bode plots by Matlab), 6.17(b), 6.19(a)-(d) (hint: for (a)-(b), also draw Nyquist plot of $G(s)$ by hand using our method and verify it by Matlab; for (d), find the range of K for $1+KG(s)$ to have all zeros stable, using our method). For 6.3(c) and (e), also figure out the gain margin and phase margin of each case at $K = 1$, for the closed-loop unity negative feedback system with open-loop transfer function $KL(s)$. Also, for each case, examine the closed-loop stability for $K = 1$.

You may use the "grid" command on the Bode plots to make it easy to figure out the numbers; note that the amplitude Bode plots from Matlab are given in db: $20\log|L(j\omega)|$, e.g., $20\log(1) = 0$, and you need to figure out the original $|L(j\omega)|$.

Homework 9 (Topic 9): 6.49, 6.50, 6.60 (hint: use $VM = \min_\omega 1/|S(j\omega)| = 1/\max_\omega |S(j\omega)|$).

Reading assignment: Sections 6.7.7 and 6.7.8 – Sensitivity.

Additional problems: (1) Draw the Nyquist plot of $G(s) = \frac{s+5}{(s-2)(s-4)}$ by hands, and find the range of K for the system $\frac{KG(s)}{1+KG(s)}$ to be stable (you may use Matlab to verify your results) (hint: use the procedure in the lecture notes). (2) Repeat the above work for $G(s) = \frac{(s+3)(s+5)}{(s-2)(s-4)}$.

Homework 10 (Topic 10): 7.17(b), 7.21, 7.22 (hint: $y = x_1$), 7.49(a), (b), (c), 9.31 (hint: from the closed-loop transfer function, put the system in the state-space controller canonical form, to get the A matrix first.)

Reading assignment: Section 9.5.2 – Lyapunov stability