Duplication Correcting Codes for live DNA Storage

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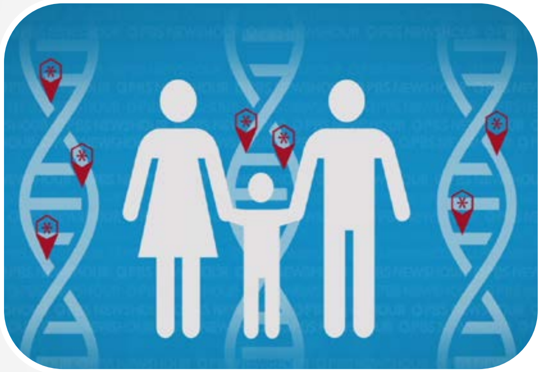
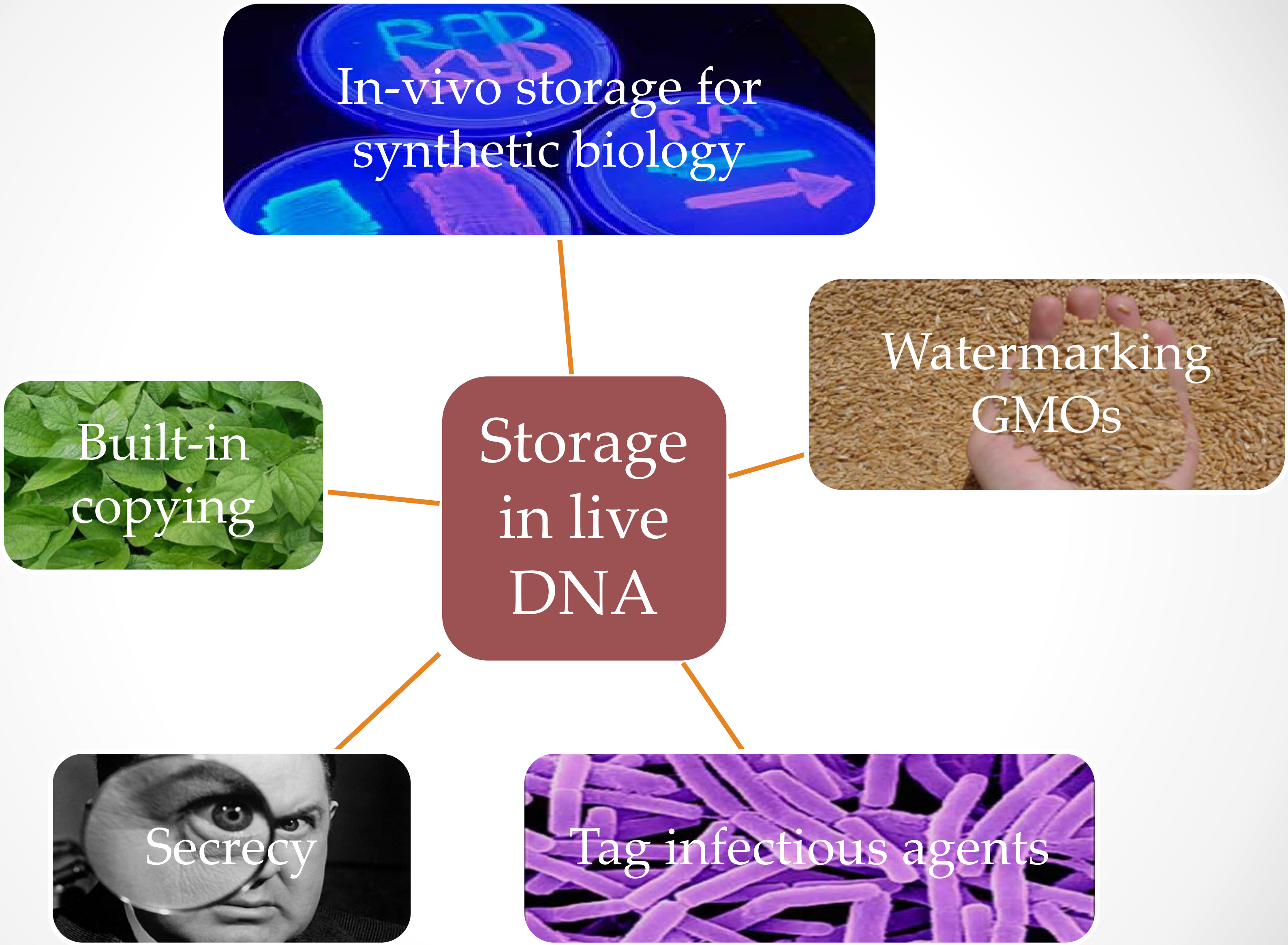
Data Storage in DNA

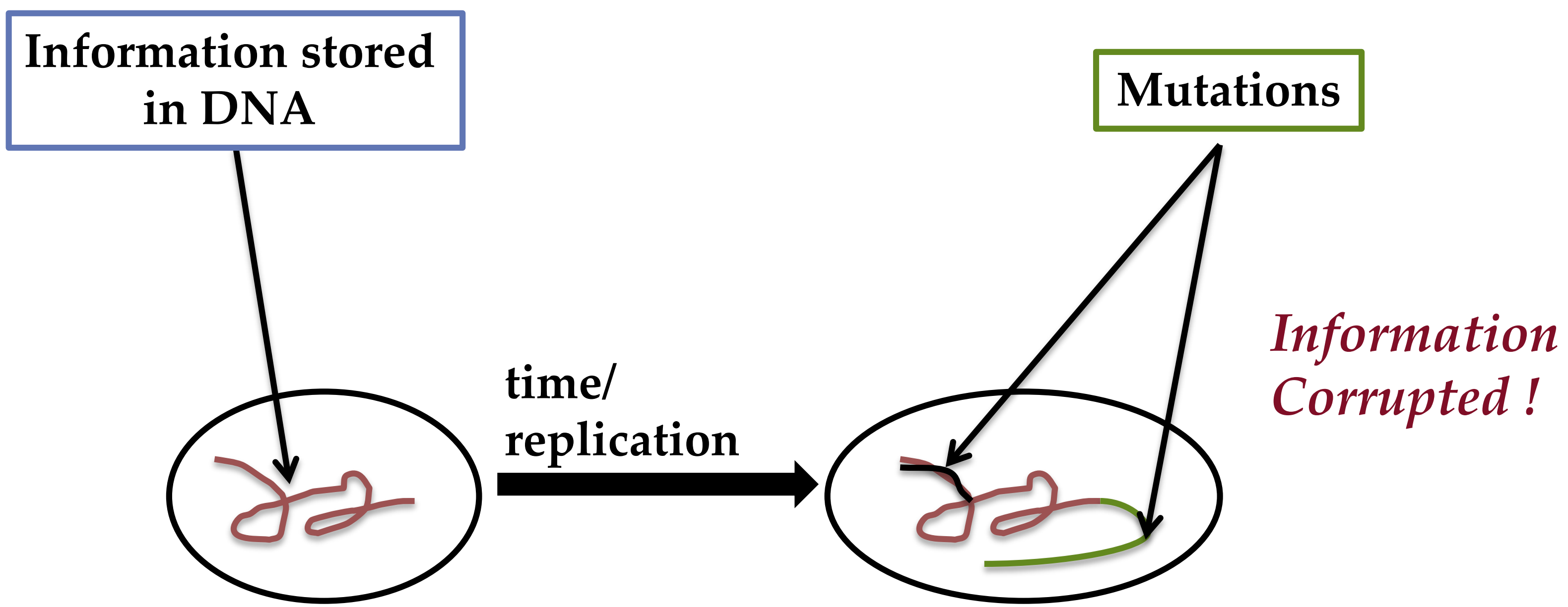


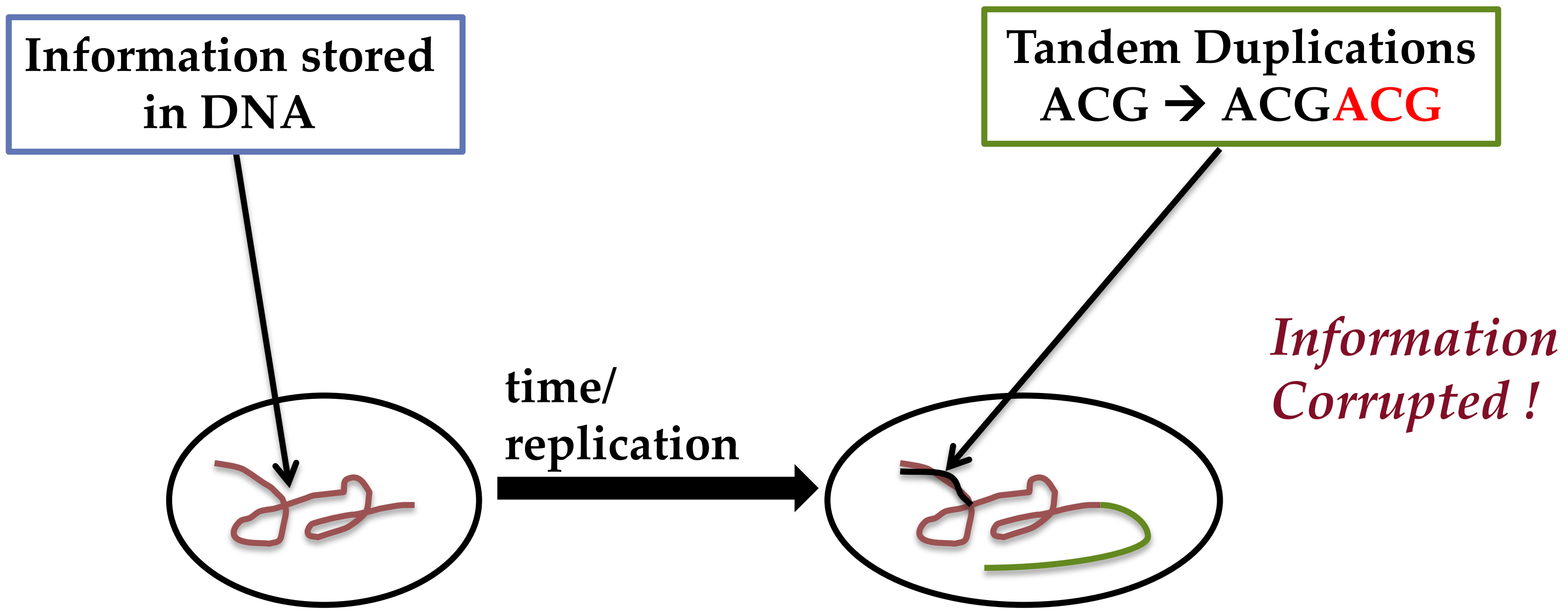
genetic information is stored in DNA

ex-vivo data storage

in-vivo data storage


In-vivo storage for synthetic biology
Watermarking GMOs
Built-in copying
Storage in live DNA
Secrecy
Tag infectious agents


Information stored in DNA
Mutations
time/ replication
Information Corrupted !


Information stored in DNA
Tandem Duplications
ACG → ACGACG
time/
replication
Information Corrupted !

Related Work

- Arita & Ohashi, 2004 – parity check bits
- Haider & Barenkow, 2007 - Hamming code or repetition code
- Yachie et. al, 2008 - copy data multiple times at different locations
- Haughton & Balado, 2013 - Coded for substitution

- Dolecek and Ananthram 2008 – Tandem duplication errors of length 1
- Mitzenmacher 2008 – Lower & upper bounds on sticky channel capacity

Tandem Duplications

AG

T_2

AGAG

T_3

AGAGGAG

T_2

AGAGGAGAG

T_7

AGAGGAGAGAGGAGAG

T_4

AGAGAGAGGAGAGAGGAGAG

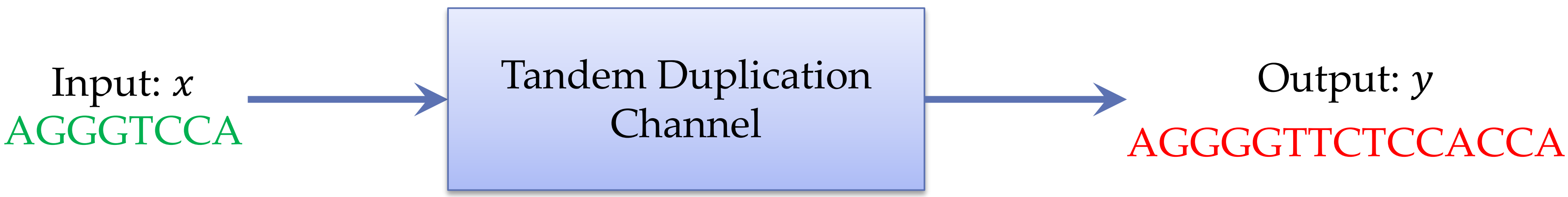
Tandem Repeats in Genome

TTTCTTTCTTTCTTTCTTTCTTTC
TTTCTTTCTTTCTTTCTTTCTTT
TTTCTTTCTTTCTTTCTTTCTT
CCTTCCTTCCTTCCTTCCTTC
TCCTTCCTTCCTTCTTTCTTT
TCTTTCTTTCTTTCTTTCTTT
TCTTTC
TCTTTC
TCTTTC

AAGAAAAAAAAAAGAAGGAGAA
GGAGAAGGGGAAGGGGAAGGGG
AAGAAGAGGAAGAGGAAGAAGA
AGAAGAAGAAGAGGAAGAAGAA
GAAGAGGAAGAAGAAAGGAGAA
AGGAGGAGAAGGAG
AGGAGAAGAAGGAGA
GGAGAAGAAGGAGAA
GAGAAGGAGAAGGGG
AGGAGAAGAAGAAGA

GGTTTGGTTTGGTTTGGTTTGG
TTTGGTTTGGTTTGGTTTGGTTT
GGTTTGGTTTGGTTTGGTTTGG
TTTGGTTTGGTTTGGTTTGGTTT
GGTTTGGTTTGGTTTGGTTTGG
TTTGGTTTGGTTTGGTTTGGTTT
GGTTTGGTTTGGTTTGGTTTGG
TTTGGTTTGGTTTGGTTTGGTTT
GGTTTGGTTTGGTTTGGTTTGG
TTTGGTTTGGTTTGGTTT

Channel Model



k-uniform Errors, T_k

Example : 2-uniform (T_2)

Input: x = ACGT

ACGT → ACG<u>CG</u>T → AC<u>AC</u>GCGT → ACACGCGT<u>GT</u>

Output: y = ACACGCGTGT

k-bounded Errors, $T_{\leq k}$

Example : 4-bounded ($T_{\leq 4}$)

Input: x = ACGT

ACGT → ACGCGT → ACGACGCGT →
AACGACGCGT → AACGACGCGTGCGT

Output: y = AACGACGCGTGCGT

Decoding by deduplication

Encoding	• Repeat-free sequences
Decoding	• Remove all repeats

Decoding by Deduplication

Removing k-uniform errors

Example : 2-uniform (T_2)

Channel output: y = ACACGCGTGT

AC~~AC~~GCGTGT → ACG~~CG~~TGT → ACGT~~GT~~

Input estimate: $\hat{x}$ = ACGT

Decoding by Deduplication

Removing k-bounded errors

Example : 4-bounded ($T_{\leq 4}$)

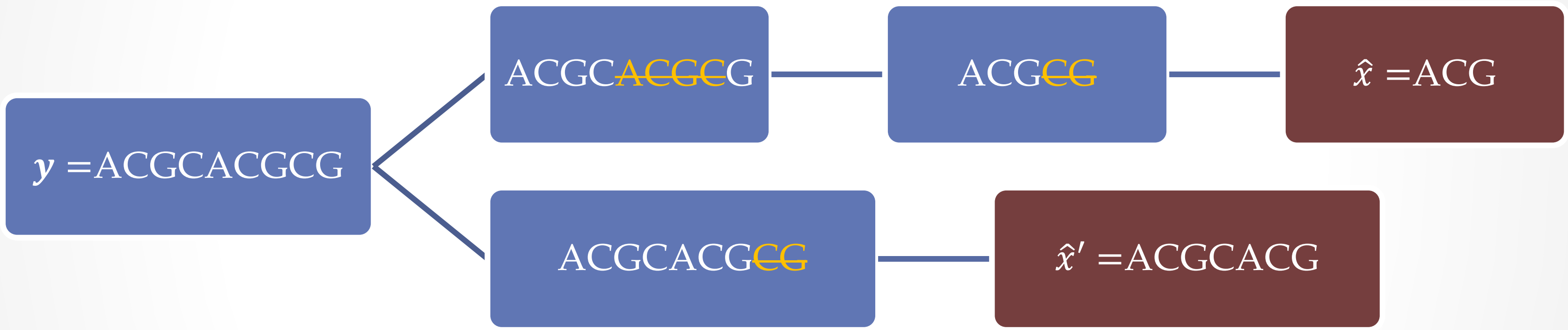
Channel output: y = AACGACGCGTGCGT

A~~A~~CGACGCGTGCGT → ACG~~ACG~~CGTGCGT → ACGCGT~~GCGT~~ → ACG~~CG~~T

Input estimate: $\hat{x}$ = ACGT

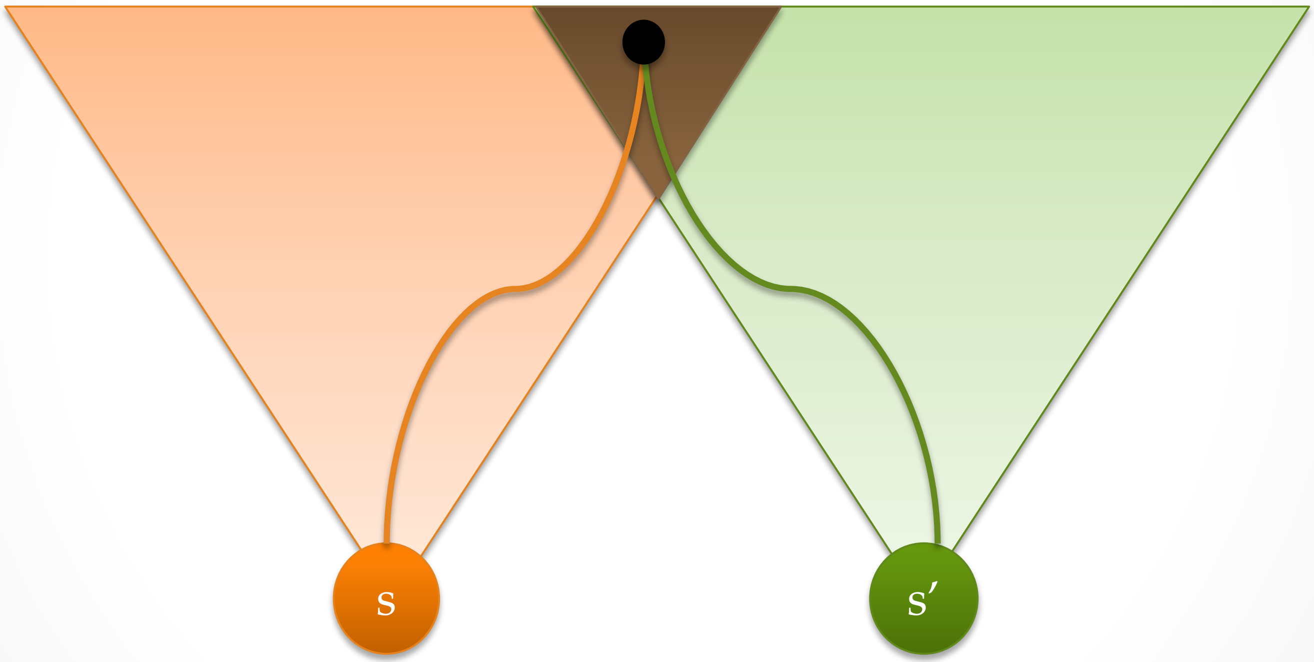
What Could Go Wrong?

Example: $T_{\leq 4}$



Root of s: repeat-free sequence that can be transformed to s via duplications

Duplication Cone



Uniqueness of Roots

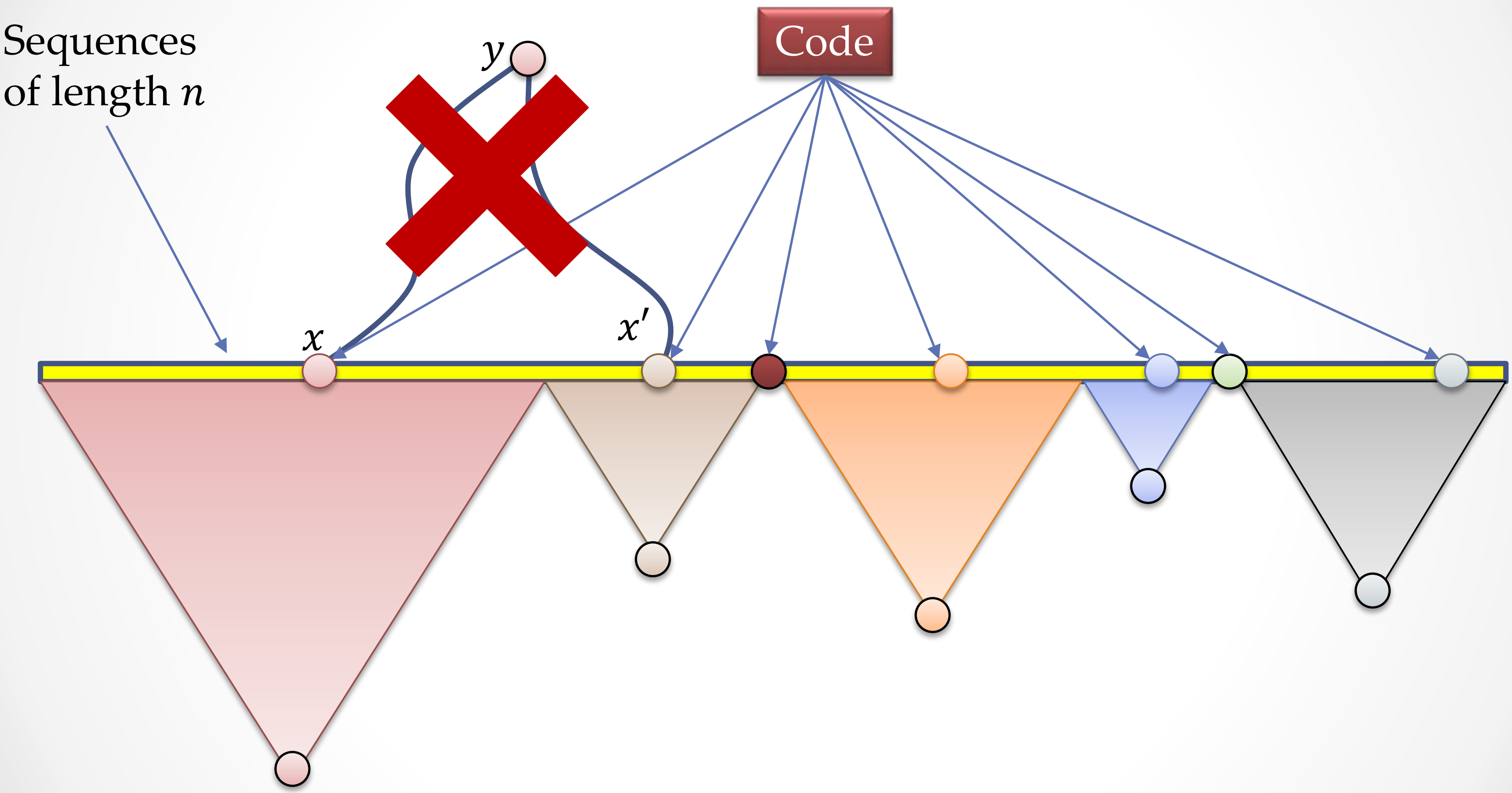
Theorem 1

For tandem duplication rule T_k, the root is unique for any k.

Theorem 2

For tandem duplication rule $T_{\leq k}$, the root is unique for $k \leq 3$.

T_k, $T_{\le 2}$, $T_{\le 3}$



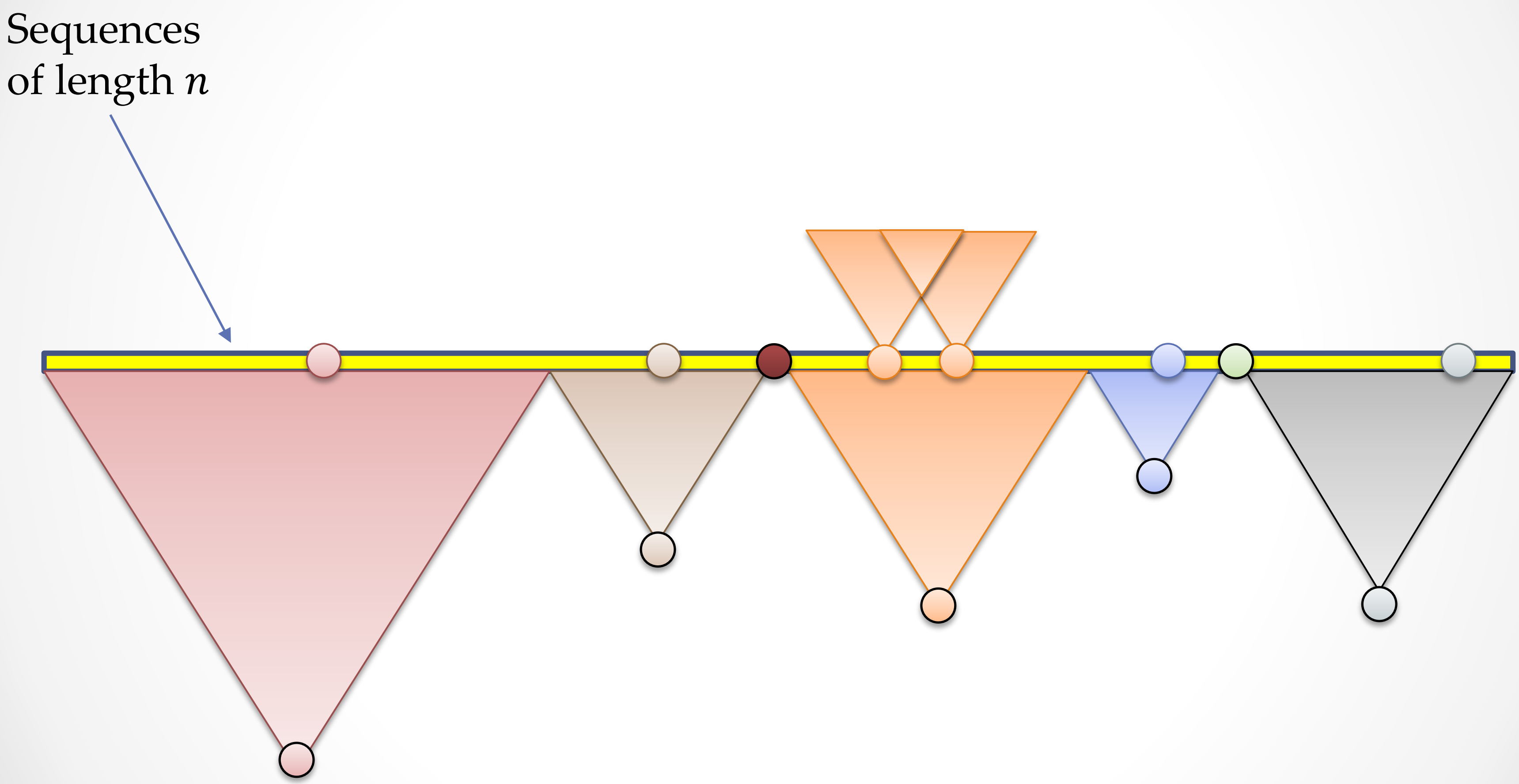
Codes for $T_k, T_{\le 2}, T_{\le 3}$ Channels

Extend each root to length n through T

Example: $T_2,\ n = 7, |\Sigma| = 4$:

ACTCTCT, AAAAAAA, CGGTATA, CATGCGA

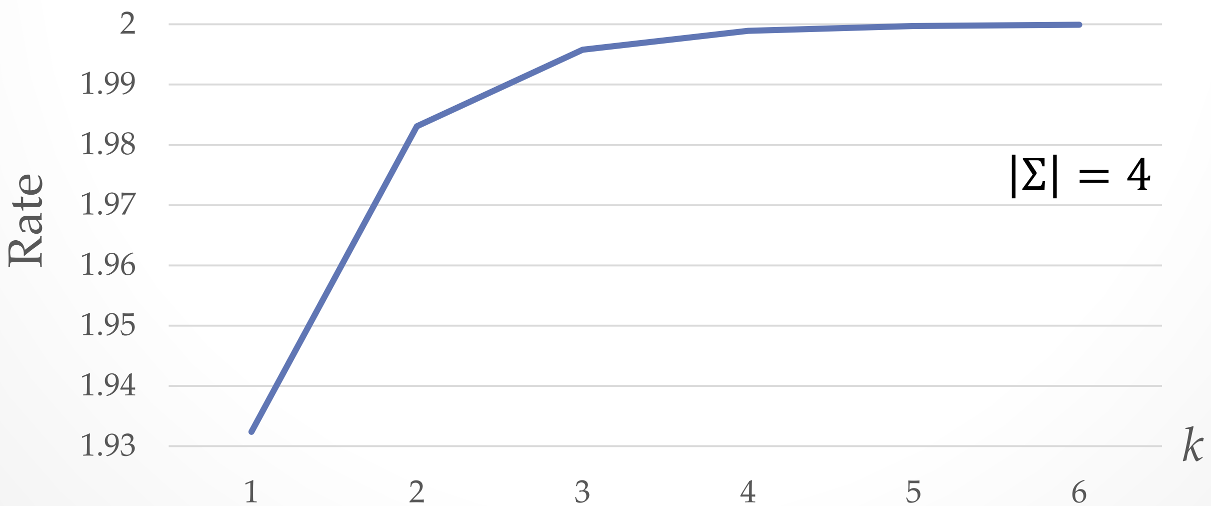
This code is optimal for T_k and $T_{\le 2}$



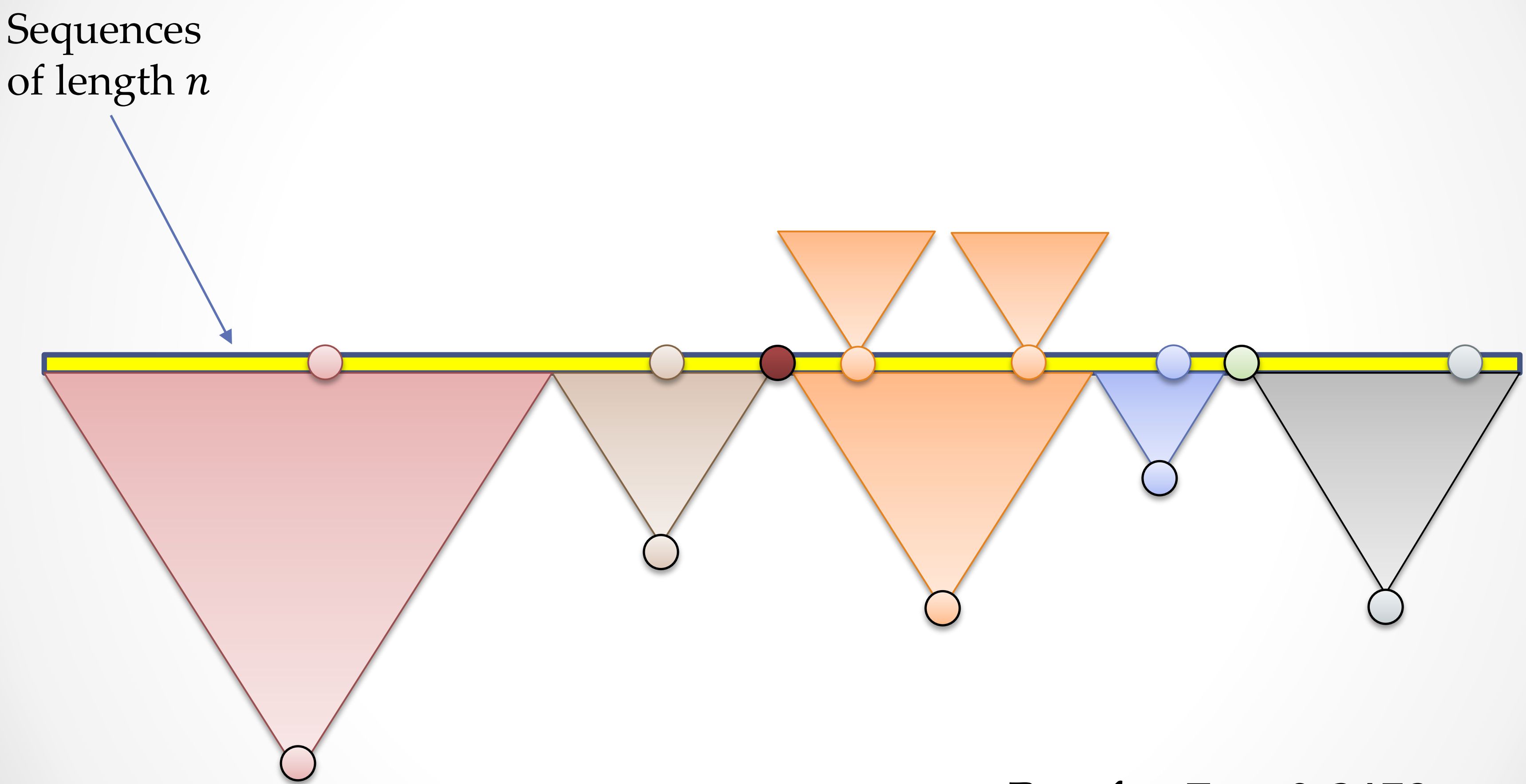
Codes for T_k Channel

Bijection: Roots $\leftrightarrow$ $\mathrm{RLL}(0, k-1)$

$$M = \sum_{i=0}^{\lfloor n/k \rfloor - 1} |\Sigma|^k M_{RLL(0,k-1)}(n-(i+1)k)$$



This code is not optimal for $T_{\leq 3}$



Rate for $T_3 \geq 0.3479$

Other Results

Construction: Optimal codes for t errors under T_k using codes in ℓ_1-metric

Theorem: Under T_U, the root is unique for all sequences if and only if

$\lvert\Sigma\rvert = 1$	$k \mid U$
$\lvert\Sigma\rvert = 2$	$U = \{k\}$ $U \supseteq \{1,2\}$
$\lvert\Sigma\rvert \geq 3$	$U = \{k\}$ $U \supseteq \{1,2\}$ $U \supseteq \{1,2,3\}$

Open Problems

- Optimal Code for ≤ 3 duplication error
- Codes for non-unique root regimes
- Codes for unbounded duplication error
- Code for duplication errors with point mutations