Problem 1. Consider the Markov chain in Fig. 1 of [Reference 1]. Assume \( m = 2 \), \( q_r = 0.5 \) and \( \lambda = 0.3 \). Solve for steady state probability \( p_0 \), \( p_1 \), and \( p_2 \).

**[Solution]**

![Markov chain diagram](image)

Easy to get \( p_{00} = 0.98 \), \( p_{11} = p_{22} = 0.5 \), \( p_{02} = 0.0194 \), \( p_{10} = 0.43 \), \( p_{12} = 0.0696 \), \( p_{21} = 0.5 \). Solve

\[
P_1P_{10} = P_0P_{02} \\
P_0P_{02} + P_1P_{12} = P_2P_{21}, \text{we obtain}
\]

\[
P_1 + P_2 + P_3 = 1
\]

\( p_0 = 0.917 \), \( p_1 = 0.0414 \), \( p_2 = 0.0414 \).

**Problem 2. Delay of Ideal Slotted CSMA/CD.** Assume that for all positive backlogs, the number of packets attempting transmission in a interval is Poisson with mean \( g \).

(a) Start with Eq. (4.42) (Reference 2), which is valid for CSMA/CD as well as CSMA. Show that for CSMA/CD,

\[
E\{t\} = \frac{\beta e^{-g} + (1 + \beta)ge^{-g} + 2\beta[1 - (1 + g)e^{-g}]}{ge^{-g}}
\]

Show that \( E\{t\} \) is minimized over \( g > 0 \) by \( g = 0.77 \), and the minimum is

\[
\min(E\{t\}) = 1 + 3.31\beta
\]

(b) Show that for this \( g \) and mean packet length 1,

\[
W = \frac{\bar{R} + \bar{y}}{1 - \lambda(1 + 3.31\beta)}
\]

(c) Evaluate \( \bar{R} \) and \( \bar{y} \) to verify the following equation for small \( \beta \)
\[ W \approx \frac{\lambda X^2 + \beta(4.62 + 2\lambda)}{2[1 - \lambda(1 + 3.31\beta)]} \] (5)

**Hint:** Here “Ideal Slotted CSMA/CD” means ideally stabilized slotted CSMA/CD. The derivation of the expected delay in this system is similar to that of Section 4.4.2 [Reference 2].

**[Solution]**

(a) Let \( E\{t\} \) be the expected time between initiations of successful packet transmissions, assuming a backlogged system with the number of transmissions after each idle slot having a Poisson distribution with mean \( g \). Using the same argument as in Eq. (4.43), but recognizing that the time occupied by a collision is now \( 2\beta \), including the idle slot after the collision, we have

\[ E\{t\} = [\beta + E\{t\}]e^{-g} + [1 + \beta]ge^{-g} + [2\beta + E\{t\}][1 - (1 + g)e^{-g}] \] (6)

\[ E\{t\} = \frac{\beta e^{-g} + (1 + \beta)ge^{-g} + 2\beta[1 - (1 + g)e^{-g}]}{ge^{-g}} = 1 + \frac{\beta(2e^{-g} - 1 - g)}{g} \] (7)

Minimizing numerically gives \( E\{t\} = 1 + 3.31\beta \) at \( g = 0.77 \)

(b) Using Little’s relation on the time average of Eq. (4.42),

\[ W = \frac{\bar{R} + \bar{\nu}}{1 - \lambda E\{t\}} = \frac{\bar{R} + \bar{\nu}}{1 - \lambda(1 + 3.31\beta)} \] (8)

(c) For small \( \beta \), the contribution of the idle and collision intervals to the residual time \( R \) can be ignored since they are proportional to \( \beta^2 \). Thus,

\[ \bar{R} \approx \frac{\lambda E\{(X + \beta)^2\}}{2} \approx \frac{\lambda X^2 + 2\lambda\beta}{2} \] (9)

As in CSMA,

\[ \bar{\nu} = E\{t\} - (1 + \beta) = 2.31\beta \] (10)

Substituting these results into the result in part (b),

\[ W \approx \frac{\lambda X^2 + \beta(4.62 + 2\lambda)}{2[1 - \lambda(1 + 3.31\beta)]} \] (11)

**Problem 3.** (a) A telephone company establishes a direct connection between two cities expecting Poisson traffic with rate 30 calls/min. The durations of calls are independent and exponentially distributed with mean 3 min. Inter-arrival times are independent of call durations. How many circuits should the company provide to ensure that an attempted call is blocked (because all circuits are busy) with probability less than or equal to 0.01? It is assume that blocked calls are
lost (i.e., a blocked call is not attempted again).

(b). Assume the number of telephones that can initiate calls is 300. Each telephone generates calls at a rate of 0.1 calls/min. The durations of calls are independent and exponentially distributed with mean 3 min. Calculate the number of circuits needed if the company should ensure a call congestion probability (as opposed to time congestion prob.) of 0.01 or less. Use the matlab program posted on the course website if necessary.

(c). Assume the number of circuits needed in (b) is \( N \). Assume there are \( 300 - N \) buffers in which calls can wait if all \( N \) servers are busy. Calculate the average waiting time. Again, use the matlab program posted on the course website if necessary.

[Solution]

(a) \( \lambda = 30, \mu = 1/3, \lambda/\mu = 90 \). Use M/M/m/m formula, we get \( m \) should be at least 107.

(b) Use Matlab program bcc.m. We find bcc(0.3,83,300)=0.009 and bcc(0.3,82,300)=0.0116. Then the number of circuits needed is at least 83.

(c) Use bcq.m. We have bcq(0.1, 1/3, 83, 300)=0.0057 min (or 0.34 sec).